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A Global Equilibrium Model of Sudden Stops and External Liquidity Management

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Abstract

Emerging market economies, which have much of their growth ahead of them, either run or should run persistent current account deficits in order to smooth consumption intertemporally. The counterpart of these deficits is their dependence on capital inflows, which can suddenly stop. We make two contributions in this paper: First, we develop a quantitative global-equilibrium model of sudden stops. Second, we use this structure to discuss practical mechanisms to insure emerging markets against sudden stops, ranging from conventional non-contingent reserves accumulation to more sophisticated contingent instrument strategies. Depending on the source of sudden stops, their correlation with world events, and the quality of the hedging instrument available, the gains from these strategies can represent a substantial improvement over existing practices.

JEL Codes: E2, E3, F3, F4, .G0, C1.

Keywords: Capital flows, sudden stops, reserves, international liquidity management, world capital markets, swaps, insurance, hedging, options, hidden states, Bayesian methods.

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1 Introduction

Emerging market economies, which have much of their growth ahead of them, either run or should run persistent current account deficits in order to smooth consumption intertemporally. The funding of these deficits is a perennial source of fragility since it requires ongoing capital inflows which can suddenly stop. While in many circumstances the breakdown in capital inflows just amplifies domestic deficiencies, there is extensive evidence that in many other cases the main culprit is not the country itself, but the international financial markets' response to shocks only vaguely related to the country's actions.

The real costs of this volatility for countries that experience open crises are extremely large. Less noticed, but at least as important in terms of their pervasiveness and cumulative impact, are the large costs paid by prudent economies. These economies do not fall into open crises but are forced to incur in a variety of costly precautionary measures and build large war-chests of international reserves, a trend that has only increased in the aftermath of the capital flow crises of the late 1990s. By now, emerging markets' reserves often exceed 20 percent of their GDP, which contrasts with the 4 to 5 percent held by developed open economies such as Australia or Canada. How effective are reserves in smoothing the impact of sudden stops unrelated to a country's actions? How much of them should be accumulated? How fast? Are there potentially less costly financial mechanisms to deal with capital flow volatility? Who would be the natural counterpart for these mechanisms? How are these mechanisms limited by financial constraints?

These are among the most pressing questions for policy-makers and researchers in emerging market economies and the international financial institutions. Unfortunately, while there has been significant conceptual progress over the last two decades in understanding some of the limitations of financial contracting with emerging markets, there has been much less progress in providing an integral framework to analyze these questions quantitatively. In this paper we take one step in this direction.

Our framework considers two representative agents, one from emerging markets (EM) and one from the developed world (W). EM's problem has two main ingredients: First, its future income is significantly higher than its current income so it would like to borrow and run persistent current account deficits. Second, it has difficulty in pledging future income to finance these deficits. The natural lender is W, but its willingness to lend varies over time in response to shocks to its pref-

erences, monitoring technology, and output. These shocks, when negative, lead to an equilibrium decline in capital flows to EM, a so called *sudden stop*.[1] Faced with this environment, EM precautions against these sudden stops. We discuss a range of options as a function of the risk-sharing markets available to EM. At a minimum, EM can accumulate noncontingent reserves, but it is often possible to improve on this common practice by adopting a hedging strategy. The costs and benefits of these strategies are endogenous in our framework and depend on the particular source of sudden stops.

In our quantitative analysis we estimate a regime switching model of sudden stops in a group of emerging markets, as implied by our theoretical framework. As part of this exercise, we also estimate the joint incidence of sudden stops with jumps in the price of assets traded in US financial markets. A calibrated version of the model matches the extent of capital flow reversals, the behavior of risk premia and the size of reserves accumulation. We then use the model to quantify the potential gains from adopting hedging strategies and conclude that, depending on the source of sudden stops, their correlation with world events, and the quality of the hedging instrument available, the gains from these strategies can be substantial. When compared to the welfare gains of existing practices, they represent improvements that range between about 10 and 115 percent.

Our paper relates to several strands of literature. The main shock that concerns us here is a sudden stop of capital inflows. The literature on sudden stops gained momentum since the Asian and Russian crises. The work of Calvo (1998) describes the basic mechanics and implications of sudden stops and Calvo and Reinhart (1999) document the pervasiveness of the phenomenon. The modelling of these sudden stops as the tightening of a financial constraint is also present in the work of Caballero and Krishnamurthy (2001), Arellano and Mendoza (2002) and Broner et al. (2003), among others.[2]

As an intermediate step in developing our substantive argument and quantifying the effects we describe, we model reserves accumulation as a buffer stock model against capital flow reversals. The view that reserves can be used to cushion the impact of external shocks exists at least since Heller (1966), was enhanced by the work on precautionary savings in macroeconomics during the

[1] See an earlier version of this paper, Caballero and Panageas (2005), for a more reduced form model, but with three agents: EM, W, and specialists. There, sudden stops are the result of shocks to specialists which EM tries to insure with W. Shocks to the monitoring technology play a similar role in the current model.

[2] See also Kiyotaki and Moore (1997) for a comprehensive model of how financial constraints are linked with cycles.

1980s and has recently returned to the fore with the large accumulation of reserves exhibited by emerging markets since the crises at the end of the 1990s (see, e.g., Lee (2004)).

Importantly, the main reason for seeking insurance and hedging in our context is not income fluctuation per-se but the potential tightening of a financial constraint. This motive parallels that highlighted by Froot et al (1993) at the level of corporations, and by Caballero and Krishnamurthy (2001) for emerging markets. While the substantive themes developed in those articles differ from ours, the basic model in our paper is in many respects a dynamic global-equilibrium version of theirs.

Closely related to our recommendations are those in the sovereign debt literature. The optimality of contingent debt and the limitations to it imposed by financial frictions are also a feature of that literature. In particular, the work of Kletzer, Newbery and Wright (1992) and Kletzer and Wright (2000), characterize feasible financial structures consistent with different degrees of commitment by a sovereign borrower and its lenders. In our model we capture financial frictions through monitoring costs, which capture features similar to those of their richer limited enforcement framework. Our paper reinforces much of the message in that literature and provides a tractable model that can be estimated and quantified.

The interaction between precautionary savings and financial constraints is also present in the closed economy framework of Aiyagari (1994). He calibrated such a model to estimates of US microeconomic income processes and other parameters and concluded that eventually agents would save enough to relax all financial constraints. In our model this does not happen because EM faces a steeper growth rate than W. Hence, EM's incentive to save in order to mitigate the effects of time varying financial frictions is counterbalanced by the reluctance to save in light of a steeply increasing endowment. The interaction of these two effects leads to a stationary level of reserves and recurrent episodes of sudden stops and consumption drops.

There has been a significant rise in volatility trades in financial markets, including the VIX (which is the index we use for our main example of hedging). The finance literature has studied the impact of such trades on the performance of hedge funds and other markets participants (see, e.g., Bondarenko (2004)). From a risk management perspective, the issues faced by these economic agents are similar to those faced by emerging market policymakers.

We setup the model in Section 2 and describe global equilibrium under different assumptions on

the risk-sharing options available. In Section 3 we estimate the sudden stop process and calibrate the model. Section 4 presents our main quantitative results and Section 5 concludes. The paper also contains several appendices.

2 A Model of Sudden Stops in Global Equilibrium

In this section we describe a model of optimal global risk-sharing in the presence of financial constraints in emerging markets. In the model, systemic crises are caused by events in world financial markets that lead to abrupt changes in capital flows to emerging markets. We start with a characterization of a perfect (sudden stop) risk sharing benchmark, and conclude with a discussion of a more realistic proxy-hedging context where available insurance instruments are only imperfectly correlated with sudden stops.

2.1 Agents and Endowments

There are two "representative" agents in the world economy, one from emerging markets (EM) and one from the developed world (W), who have common preferences

$$\mathrm{E}_t\left[\int_t^\infty A_s \frac{(C_s^j)^{1-\gamma}}{1-\gamma} e^{-\rho(s-t)}\, ds\right] \text{ for } j \in \{W, EM\} \tag{1}$$

The parameter ρ represents the discount rate, $\gamma > 1$ the coefficient of relative risk aversion, C_s^j the consumption of agent j, and A_s captures a taste shock, whose properties will be specified later.

The world is an endowment economy. EM's current income is low relative to its future income (it has yet to catch up), which we capture through a pre-development phase (which is the focus of the paper) and a post-development phase. During the former, EM's endowment is described by $\Lambda_t Y_t$, where Λ_t is a process that can take either the value 1 or the value $\Lambda^d \leq 1$ (we shall specify the exact dynamics of these transitions shortly), and Y_t is given by

$$\frac{dY_t}{Y_t} = \mu_t dt + \sigma dB_t. \tag{2}$$

where $\mu_t, \sigma > 0$ and dB_t is a standard Brownian motion. EM (permanently) transits from the pre- to the post-development phase at an exponentially distributed random time τ^G, with constant hazard g. This catching-up transition is independent of all other sources of uncertainty in the

model. EM's post-development income is equal to $\kappa\Lambda_t Y_t$, where $\kappa > 1$. W's endowment is given by $\beta\Lambda_t Y_t$, where

$$\beta \geq \kappa > 1 \tag{3}$$

for all $t \geq 0$.

2.2 Growth-Contingent Debt and Capital Flows

Since in the pre-development phase EM's expected growth rate is higher than that of W, it is natural for EM to borrow from W. Moreover, given the stochastic nature of catching up in our model, optimal debt contracts are contingent on the transition to development. Concretely, (growth-contingent) debt contracts establish that W provides a flow of resources $f_t p_t Y_t$ to EM over the next infinitesimal time interval dt, in exchange for receiving a promise to a stream of payments $f_t \kappa Y_s$ for all $s > t$, if development arrives in the interval dt and 0 otherwise. The price of these growth contingent debt contracts is p_t while f_t is the number of contracts (expressed as a fraction of Y_t). By using these contracts, EM can exchange a stream of post-development for pre-development income.

In the absence of frictions, growth-contingent debt contracts are claims that complete the market with respect to the uncertainty introduced by the random arrival of development, and ensures that the marginal utilities of W and EM are always proportional to each other, both pre-and post development.[3]

Since there is only one source of uncertainty in the model in the post-development phase, assuming that EM and W are able to trade in W's equities implies that W and EM face complete markets after τ^G.[4] To simplify the analysis, we will assume this trade is feasible (after τ^G) and use

[3] Note that these contingent contracts can be implemented through equity trades. For this, assume that there exist two equities that give the owner an entitlement to the endowment of EM and W respectively. Let their prices be P_t^{EM} and P_t^W respectively and their dividends be Y_t and βY_t pre-development and κY_t and βY_t post development. To produce the payoffs of a growth contingent contract, one needs to purchase $\widehat{x}_t$ units of EM's equity financed by $\widehat{x}_t P_t^{EM}/P_t^W$ units of W's equities. The holder of such a claim delivers a stream of payments equal to $\widehat{x}_t Y_t \left[1 - \left(P_t^{EM}/P_t^W\right)\beta\right]$ pre-development and receives a capital gain equal to $\widehat{x}_t \left(\left[P_{\tau^G+}^{EM} - \left(P_{\tau^G-}^{EM}/P_{\tau^G-}^W\right) P_{\tau^G+}^W\right]\right)$ once the transition to development takes place. By arbitrage $P_{\tau^G+}^{EM} = (\kappa/\beta) P_{\tau^G+}^W$ and hence the capital gain can be expressed as $\widehat{x}_t \left(\left[1 - \left(P_{\tau^G-}^{EM}/P_{\tau^G-}^W\right)(\kappa/\beta)\right]\right) P_{\tau^G+}^{EM}$. Since $\widehat{x}_t$ is arbitrary, one can let $\widehat{x}_t = 1/\left(\left[1 - \left(P_{\tau^G-}^{EM}/P_{\tau^G-}^W\right)(\kappa/\beta)\right]\right)$. Hence, by purchasing $\widehat{x}_t$ units of EM's shares financed by $\widehat{x}_t \frac{P_t^{EM}}{P_t^W}$ units of W's shares, one can create one unit of a growth contingent contract. In summary, a growth contingent contract is equivalent to a *carry trade* in equities.

[4] See e.g. Duffie and Huang (1985).

P_t^W to denote the price of W's shares and S_t^{EM}, S_t^W to denote the holdings of such shares by EM and W respectively, with $S_t^{EM} + S_t^W = 1$.

2.3 Normal Times, Crises, and Sudden Stops

During the pre-development phase, there are two states of nature s_t: "normal" ($s_t = 0$) and "crisis" ($s_t = 1$) times. The transition hazards from the former to the latter and vice versa are λ and $\widetilde{\lambda}$, respectively. For simplicity, there are no crises after the time of development τ^G. (i.e., $s_t = 0$ for all $t > \tau^G$).

We assume that for each growth contingent contract f_t, W pays a monitoring cost equal to $i_t Y_t$, where

$$i_t = \begin{cases} \bar{i} & \text{if } s_t = 0 \\ \bar{i} + \overline{q} & \text{if } s_t = 1 \end{cases} \tag{4}$$

We shall assume throughout that $\bar{i} > 0$ and $\overline{q} \geq 0$. These monitoring costs capture the idea that growth contingent debt effectively represents uncollateralized lending to EM, which in practice is subject to a series of incentive problems that are costly to overcome. For our purposes, these monitoring costs inhibit intertemporal consumption smoothing between pre-and post-development times. Moreover, we assume that during normal times $\Lambda_t = 1$, while during crises $\Lambda_t = \Lambda^d \leq 1$. Hence, crises are also times when world output may fall below the (stochastic) trend Y_t.

To produce sudden stops within our framework, we assume that

Assumption 1 *Either* $\overline{q} > 0$ *or* $\Lambda^d < 1$ *(or both)*

Assumption 1 requires that at least one of two events happens during a crisis: Either the cost of monitoring increases, or world output declines below its trend, so that it becomes costlier to sacrifice consumption in monitoring activities.

For the purposes of calibrating the model and performing robustness checks later on, it is also useful to allow for two extra layers of generality. First, the growth rate μ_t could take different values during crises, normal times, and development:

$$\mu_t = \begin{cases} \mu_0 + s_t (\mu_1 - \mu_0) & \text{if } t < \tau^G \\ \mu^G & \text{if } t \geq \tau^G \end{cases}$$

Second, we assume that the preference shock process A_t in equation (1) is given by

$$A_t = \left(\overline{A}\right)^{N_t}$$

where $\overline{A} \geq 1$ and N_t is the number of all transitions from normal times to crises that have have happened until time t. In the special case where $\overline{A} = 1$, the objective (1) becomes a standard expected utility maximization problem.

2.4 Perfect (Sudden Stop) Risk Sharing Benchmark

During normal times EM precautions against sudden stops. We assume that there are two potential contracts that can be used for that purpose. The first kind are non-contingent bonds (reserves), which are zero net supply claims that pay an interest rate r_t per unit of time dt. Second, agents also have access to (sudden stop) hedging contracts. For each of these contacts, EM delivers a payment of $\pi_t Y_t$ to W per unit of time dt while $s_t = 0$, where π_t is the "price" of such a contract. If s_t stays at 0 over the interval dt, then W makes no transfers to EM and the contract is terminated. However, if s_t jumps to 1 over the next time interval dt, EM receives a flow of Y_t (per contract) from W until s_t transits back to 0.

Hedging contracts are hence similar in nature to contingent debt contracts, although in this case EM delivers the payments to W *first*, and W has to repay when a transition to a sudden stop happens. We assume that W has full commitment to such contracts and hence there is no risk of W defaulting on EM.

In summary, sudden stop contracts represent forms of insurance that can help EM hedge against both the *occurrence* and the *duration* of sudden stops.

2.4.1 Net Asset processes

We can now describe the dynamic budget constraints for EM and W. Letting X_t^j denote j's assets (reserves), f_t the amount of contingent debt as a share of Y_t, and n_t the number of sudden stop contracts as a share of Y_t, we have:

1. (Pre-development, $s_t = 0$) If $t \leq \tau^G$ and $s_t = 0$, EM's net asset process is

$$dX_t^{EM} = \left[r_t X_t^{EM} - C_t^{EM} + \left(1 + f_t p_t - n_t \pi_t\right) Y_t\right] dt \tag{5}$$

with r_t being the prevailing interest rate, and W's asset process is

$$dX_t^{W} = \left[r_t X_t^{W} - C_t^{W} + \left(\beta - f_t \left(p_t + \bar{i}\right) + n_t \pi_t\right) Y_t\right] dt \tag{6}$$

2. (Pre-development, $s_t = 1$) If $t \leq \tau^G$ and $s_t = 1$, EM's net asset process is

$$dX_t^{EM} = \left[r_t X_t^{EM} - C_t^{EM} + \left(\Lambda^d + f_t p_t + n_\tau\right) Y_t\right] dt \tag{7}$$

with τ being the last time at which s_t jumped from 0 to 1

$$\tau = \sup_{u \leq t}\{s_u = 0\}.$$

The corresponding W's asset process is

$$dX_t^W = \left[r_t X_t^W - C_t^W + \left(\beta\Lambda^d - f_t\left(p_t + \bar{i} + \bar{q}\right) - n_\tau\right) Y_t\right] dt \tag{8}$$

3. (Post-development) If $t > \tau^G$ then

$$dX_t^{EM} = \left[r_t X_t^{EM} - C_t^{EM} + \left(1 - f_{\tau^G}\right)\kappa Y_t\right] dt + S_t^{EM} dP_t^W \tag{9}$$

$$dX_t^W = \left[r_t X_t^W - C_t^W + \left(\beta + \kappa f_{\tau^G}\right) Y_t\right] dt + S_t^W dP_t^W \tag{10}$$

Let us now summarize our setup. Equations (5) and (6) describe the evolution of assets prior to development ($t < \tau^G$) whenever $s_t = 0$. In this regime EM has access to three markets: it "borrows" f_t units of growth-contingent debt, invests in n_t hedging contracts, and also invests in noncontingent bonds. The processes p_t, π_t and r_t are the respective prices in these markets.

Equations (7) and (8) are similar to (5) and (6), except that the economy is in a regime where $s_t = 1$ and hence the n_τ hedging contracts that EM entered when s_t jumped to 1 are delivering the promised payoffs to EM. Finally, equations (9) and (10) state that after EM's transition into development, EM has to deliver payoffs to W as specified by the outstanding growth contracts at the time of transition into development (f_{τ^G}).

Finally, we assume that if EM enters short positions in non-contingent bonds, then W pays $i_t Y_t \min[X_t^{EM}, 0]$ monitoring costs per unit of time dt. This assumption implies that growth contingent contracts and short positions in non-contingent bonds are subject to the same monitoring costs, since they both present forms of borrowing.

2.4.2 Global Equilibrium

We are now able to define an equilibrium in the global economy:

Definition 1 *An equilibrium is a collection of (progressively measurable) processes* C_t^W, C_t^{EM}, f_t, n_t, π_t, p_t, r_t, X_t^{EM}, X_t^W, S_t^{EM}, S_t^W, P_t^W *such that*

1. *Given the price processes* r_t, π_t, p_t, P_t^W *the quantity processes* C_t^{EM}, X_t^{EM}, f_t, n_t *and* S_t^{EM} *maximize (1) (for* $j = EM$*) subject to (5), (7) and (9).*

2. *Given the price processes* r_t, π_t, p_t, P_t^W *the quantity processes* C_t^W, X_t^W, f_t, n_t *and* S_t^W *maximize (1) (for* $j = W$*) subject to (6), (8) and (10).*

3. *Markets clear, i.e.:*

$$X_t^{EM} + X_t^W = 0 \text{ for all } t \tag{11}$$

$$C_t^{EM} + C_t^W = \begin{cases} Y_t \left[(1+\beta)\left(1 + s_t(\Lambda^d - 1)\right) - i_t f_t \right] & \text{if } t \leq \tau^G \\ Y_t \left(\kappa + \beta\right) & \text{if } t > \tau^G \end{cases} \tag{12}$$

$$S_t^{EM} + S_t^W = 1 \text{ for all } t > \tau^G \tag{13}$$

Remark 1 *Note that all remaining financial markets clear by construction, since equations (5)-(10) recognize the zero net supply nature of the markets for hedging contracts and growth contingent contracts.*

The definition of equilibrium is standard. It requires that all actions should be optimal given prices and that all markets clear. Next we construct an equilibrium and characterize its properties. Before proceeding, it will be useful to define

$$x_t^i = \frac{X_t^i}{Y_t}, \text{ where } i = \{W, EM\},$$

$$c_t^i = \frac{C_t^i}{Y_t}, \text{ where } i = \{W, EM\}.$$

The following proposition constructs an equilibrium and outlines some properties of pre-development allocations and prices. (Post-development allocations and prices are given in the appendix).

Proposition 1 *For appropriate constants* $c^{0,W}, f^0, f^1$ *there exists an equilibrium with the following properties:*

1. $X_t^{EM} = X_t^W = 0$ *for all* t.

2. *The consumption process for W is given by*

$$c_t^W = \begin{cases} c^{0,W} \text{ if } s_t = 0 \\ c^{1,W} \text{ if } s_t = 1 \end{cases} \tag{14}$$

where

$$c^{1,W} = \frac{\Lambda^d (\beta + 1) - \left(\bar{i} + \bar{q}\right) f^1}{\beta + 1 - \bar{i} f^0} c^{0,W} \tag{15}$$

and the consumption process for EM is given by (12) after plugging in for C_t^W.

3. *Let the constants* ϖ *and* ν *be defined as*

$$\begin{aligned} \varpi &\equiv \frac{1}{\rho + \widetilde{\lambda} - (1-\gamma)\left(\mu_1 - \gamma\frac{\sigma^2}{2}\right)}, \\ \nu &= \rho - (1-\gamma)\left(\mu_G - \gamma\frac{\sigma^2}{2}\right). \end{aligned} \tag{16}$$

and assume that $\nu < 1$. *When* $s_t = 0$ *the prices* r_t, p_t, π_t *are given by the constants*

$$r^0 \equiv \rho + \left(\mu_0 - \frac{\sigma^2}{2}\right)\gamma - \frac{(\gamma\sigma)^2}{2} - \lambda\left[\overline{A}\left(\frac{c^{1,W}}{c^{0,W}}\right)^{-\gamma} - 1\right] - g\left[\left(\frac{\beta + \kappa f^0}{c^{0,W}}\right)^{-\gamma} - 1\right] \tag{17}$$

$$\pi = \lambda\varpi\overline{A}\left(\frac{\Lambda^d (\beta + 1) - \left(\bar{i} + \bar{q}\right) f^1}{\beta + 1 - \bar{i} f^0}\right)^{-\gamma} \tag{18}$$

$$p^0 = g\frac{\kappa}{\nu}\left(\frac{\beta + \kappa f^0}{c^{0,W}}\right)^{-\gamma} - \bar{i} \tag{19}$$

while in the state $s_t = 1$ *the prices* r_t *and* p_t *are given by the constants*

$$r^1 \equiv \rho + \left(\mu_1 - \frac{\sigma^2}{2}\right)\gamma - \frac{(\gamma\sigma)^2}{2} - \widetilde{\lambda}\left[\left(\frac{c^{0,W}}{c^{1,W}}\right)^{-\gamma} - 1\right] - g\left[\left(\frac{\beta + \kappa f^1}{c^{1,W}}\right)^{-\gamma} - 1\right] \tag{20}$$

$$p^1 = g\frac{\kappa}{\nu}\frac{\left(\beta + \kappa f^1\right)^{-\gamma}}{\left(c^{1,W}\right)^{-\gamma}} - \left(\bar{i} + \bar{q}\right) \tag{21}$$

There are two results in Proposition 1 that are worth highlighting: First, in general equilibrium there is no accumulation of noncontingent bonds ($X_t^{EM} = X_t^W = 0$). EM only uses contingent contracts to precaution against the arrival of the state $s_t = 1$. Given the anticipated steep increase in EM's income post development, any form of pre-development consumption sacrifice to hedge against a sudden stop is costly. Since reserves are noncontingent, they imply that EM is saving resources for all states of the world – even states of the world where no sudden stop takes place.

This makes reserves too costly as a hedging instrument compared to claims that only deliver payoffs exclusively in states of the world where such payoffs are needed (i.e. during sudden stops).

Second, Proposition 1 states that unconstrained trading in hedging contracts completes markets with respect to the arrival of state $s_t = 1$. To see this, note that (15) together with (12) implies that

$$\left(\frac{C^W_{\tau^+}}{C^W_{\tau^-}}\right)^{-\gamma} = \left(\frac{\Lambda^d(\beta+1) - \left(\bar{i}+\bar{q}\right) f^1}{\beta + 1 - \bar{i} f^0}\right)^{-\gamma} = \left(\frac{C^{EM}_{\tau^+}}{C^{EM}_{\tau^-}}\right)^{-\gamma} \tag{22}$$

where τ^- refers to an instant before the arrival of $s_t = 1$ and τ^+ to the instant thereafter. (Formally, $s_{\tau^-} = 0$, and $s_{\tau^+} = 1$). The first equality in (22) follows from (15), while the second equality follows from the goods market clearing condition (12).

A simple limit case ($\beta \to \infty$) When W becomes very large relative to EM, we can sharpen the results further:

Lemma 1 *As $\beta \to \infty$ the solution to the system (80)-(82) satisfies $c^{0,W}/\beta \to 1$ and*

$$p^0 = g\frac{\kappa}{\nu} - \bar{i}, \quad p^1 = g\frac{\kappa}{\nu}\left(\Lambda^d\right)^{\gamma} - \left(\bar{i}+\bar{q}\right) \text{ and } \pi = \lambda\varpi\overline{A}\left(\Lambda^d\right)^{-\gamma} \tag{23}$$

f^0 is given by

$$f^0 = \frac{\kappa - \left[\frac{p^0}{g\frac{\kappa}{\nu}}\right]^{-1/\gamma}\left[1 + p^1\frac{\pi}{1+\pi\Lambda^d}\left(\Lambda^d - \left(\frac{p^0}{p^1}\right)^{\frac{1}{\gamma}}\right)\right]}{\kappa + \frac{1}{1+\pi\Lambda^d}\left[\frac{p^0}{g\frac{\kappa}{\nu}}\right]^{-1/\gamma}\left(p^0 + p^1\pi\left(\frac{p^0}{p^1}\right)^{\frac{1}{\gamma}}\right)} \tag{24}$$

and

$$f^1 = 1 - \frac{1}{\Lambda^d}\left(\frac{p^0}{p^1}\right)^{\frac{1}{\gamma}}\left(1 - f^0\right) \tag{25}$$

Furthermore $f^0 > f^1$, and since both f^0 and f^1 have finite limits, the term $\left(\frac{(\beta+1)\Lambda^d - \left(\bar{i}+\bar{q}\right)f^1}{\beta+1-\bar{i}f^0}\right)^{-\gamma}$ in equation (22) converges to 1 and hence

$$\frac{C^{EM}_{\tau^+}}{C^{EM}_{\tau^-}} = \Lambda^d.$$

Lemma 1 implies that when W is large compared to EM, the only shocks that can make EM's consumption drop when entering a sudden stop are world-wide cyclical shocks. Hence, the extra drop in available resources to EM that is caused by capital flow reversals is smoothed out. To see

this most clearly, it is instructive to focus on the special case where sudden stops are caused purely by jumps in the monitoring costs i_t, and there are no cyclical variations in output, so that $\Lambda^d = 1$. Then Lemma 1 implies that EM's consumption experiences no change upon entering a sudden stop.

In order to understand this result, note that since $\Lambda^d = 1$, $\frac{C^{EM}_{\tau^+}}{C^{EM}_{\tau^-}} = 1$ and $X^{EM}_t = 0$, equations (5) and (7) imply that

$$n = \frac{\left(f^0 p^0 - f^1 p^1\right)}{(1+\pi)}. \tag{26}$$

Equation (26) states that EM buys an amount of hedging contracts that is proportional to the decline in capital flows $\left(f^0 p^0 - f^1 p^1\right)$ upon entry into a sudden stop. Additionally, as $\beta \to \infty$, equation (19) implies that $p^0 \to g\frac{\kappa}{\nu} - \bar{i}$ and equation (21) implies that $p^1 \to g\frac{\kappa}{\nu} - \left(\bar{i} + \overline{q}\right)$, so that $p^0 > p^1$. Since $f^0 > f^1$ it follows that $n > 0$. The fact that $n > 0$ implies that adjustments of prices are not enough by themselves to attain consumption smoothing, and there is non-zero trading.[5]

One might suspect that when consumption of EM becomes perfectly smooth, then sudden stop insurance must be actuarially fair. This is not true however. The easiest way to see this is to allow $\overline{A} > 1$. In this case the value (in state $s = 0$) of obtaining a unit of consumption once s switches from 0 to 1 is given by

$$\lambda\overline{A}\left(\frac{C^W_{\tau^+}}{C^W_{\tau^-}}\right)^{-\gamma} = \lambda\overline{A}\left(\frac{\Lambda^d(\beta+1) - \left(\bar{i}+\overline{q}\right)f^1}{\beta + 1 - \bar{i}f^0}\right)^{-\gamma}$$

When $\beta \to \infty$, and $\Lambda^d = 1$, the above term becomes $\lambda\overline{A} > \lambda$. Hence sudden stop insurance is not actuarially fair. Nevertheless, consumption is not affected by a sudden stop (although marginal utility is).

More importantly for our purposes, note that EM prefers to use hedging rather than hold any reserves, no matter how large is $\overline{A}$ (or Y^d for that matter). The reason is that reserves "force" EM to purchase insurance in all states of the world - both the states that command a high risk premium and those that do not. By contrast, hedging always allows EM to isolate the states of the world where it needs insurance (i.e. when a sudden stop occurs). Since EM only needs insurance in these states of the world and wants to avoid sacrificing pre-development consumption, hedging is always a preferred precautionary measure.

[5] This is in contrast to many examples of endowment economies in macroeconomics and asset pricing, where prices adjust in such a way that agents are happy to just hold their endowments without trading. Here there is active trading in both growth-contingent debt *and* sudden stop hedging contracts.

More generally, when β is finite, even though sudden stop contracts cannot eliminate the presence of sudden stops, they can substantially mitigate their effects on consumption.

2.5 Imperfect (Sudden Stop) Risk Sharing

In practice there is a variety of impediments to perfect risk sharing, including agency, behavioral, and liquidity problems. In this section we capture some of these by assuming that sudden stop risk sharing is incomplete along dimensions that seem realistic. For the most part, central banks hold their reserves entirely in US and Euro treasuries rather than investing in hedging instruments. Thus, later on we calibrate our model using this extreme noncontingent scenario. However, in our analysis in this section we also introduce assets whose payoffs are correlated —although not perfectly— with sudden stop arrivals. This extension allows us to discuss practical improvements to current central banks' practices in emerging market economies.

Concretely, we preserve the environment developed up to now, with one exception: Perfect risk sharing instruments no longer exist but instead there is an asset with a payoff that is imperfectly correlated with the occurrence of sudden stops. As one would expect, the optimal portfolio of the country will now include both, shares of this asset and noncontingent reserves. Among other factors, the particular composition of the portfolio depends on the degree of correlation of the asset with sudden stops, on the level of reserves, and on the sources of sudden stops and their general equilibrium implications.

2.5.1 Danger zones and net asset processes

In order to introduce the hedging asset we decompose the arrival of sudden stops into two steps. First, there is a Poisson process with intensity $\widehat{\chi}$ that puts EM at danger of a sudden stop. These danger zones take place at random times τ_i^D, $i = 1, 2, 3....$, which we generically denote by τ^D instead of τ_i^D. At these times τ^D, the country enters a sudden stop (i.e. the state $s_{\tau_+^D}$ becomes 1) with probability $P(s_{\tau_+^D} = 1)$ and avoids it with probability $P(s_{\tau_+^D} = 0) = 1 - P(s_{\tau_+^D} = 1)$. To preserve the earlier environment we require throughout that

$$\lambda = \widehat{\chi} P(s_{\tau_+^D} = 1). \tag{27}$$

Let us now assume that there is a financial asset with payoff F_t, that also has the potential to

exhibit a jump at danger times τ^D. Using J to denote a jump, we have that F_t exhibits jumps according to a Poisson process with intensity:

$$\lambda_J \equiv \widehat{\chi} P(J=1) \tag{28}$$

The critical assumption in this section is that there is some correlation, although imperfect, between sudden stops and jumps in the payoff F_t: $P(J=1, s_{\tau_+^D}=1) \in (0,1)$.

Letting Φ_t denote a counting process that increases by 1 at times τ^D, and λ_t^* denote a required rate of return (in excess of the interest rate) for holding F_t, the price of asset F_t follows the process

$$\frac{dF_t}{F_t} = r_t dt + \left(J d\Phi_t - \lambda_t^* dt\right). \tag{29}$$

Without loss of generality, we can condition on those times τ^D where we observe *either* a jump $(J=1)$ *and/or* a transition into SS$\left(s_{\tau_+^D}=1\right)$.[6] Hence from now on let us define:

$$\chi \equiv \widehat{\chi}(1 - P(s_{\tau_+^D}=0, J=0)) \tag{30}$$

which is the hazard rate for observing either a jump in J or a transition into $s_{\tau_+^D}=1$. An obvious corollary is that there are only three possible outcomes that can take place at those times, namely: $\left(s_{\tau_+^D}=1, J=1\right)$, or $\left(s_{\tau_+^D}=0, J=1\right)$, or $\left(s_{\tau_+^D}=1, J=0\right)$. Let $p_{J=1,s=1}$, $p_{J=1,s=0}$ and $p_{J=0,s=1}$ denote the respective (conditional) probabilities of these three events.

We will not consider the possibility of insuring the duration of the sudden stop, and simplify the analysis by assuming that, conditional on entering a sudden stop, the transition out of it is independent of any jumps in J and happens with intensity $\widetilde{\lambda}$.

The presence of a risky asset with the above properties modifies the analysis in only two respects. First, the evolution of X_t^{EM} becomes:

$$dX_t^{EM} = \left\{ r_t X_t^{EM} - \lambda_t^* \xi_t F_t - C_t^{EM} + \left[1 + (\Lambda^d - 1)s_t + f_t p_t\right] Y_t \right\} dt + \xi_t F_t J d\Phi_t \tag{31}$$

where ξ_t is the dollar amount invested in the risky asset F_t. Since the asset F_t is in zero net supply, the equivalent dynamic evolution equation for X_t^W is

$$dX_t^W = \left\{ r_t X_t^W + \lambda_t^* \xi_t F_t - C_t^W + \left[\beta + (\Lambda^d - 1)s_t - f_t p_t\right] Y_t - f_t Y_t \left(\overline{i} + s_t \overline{q}\right) \right\} dt - \xi_t F_t J d\Phi_t \tag{32}$$

[6] The reason is that the value function of neither agent (W or EM) experiences any change when $s_{\tau_+^D}=0$ and $J=0$. Hence the times when $s_{\tau_+^D}=0$ and $J=0$ are irrelevant for equilibrium allocations, prices and welfare.

As with reserves, we require that $\xi_t \geq 0$, so that EM has to *pay upfront* for the F_t contracts that it purchases.[7] Finally, also note that while equations (31) and (32) hold in both states $s_t = 0$ and $s_t = 1$, in equilibrium it turns out that $\xi_t = 0$ when $s_t = 1$, since the asset F_t has no hedging value in that state.

2.5.2 Global equilibrium with imperfect risk sharing

The equilibrium definition in the presence of asset F_t is identical to definition 1 with the obvious modification that now the consumers optimally choose ξ_t instead of n_t. We do not repeat it here for brevity. The next proposition shows how to construct an equilibrium in the presence of imperfect hedging contracts

Proposition 2 *Define*

$$x_t = \frac{X_t^{EM}}{Y_t}, \widetilde{\xi}_t = \frac{\xi_t F_t}{Y_t} \tag{33}$$

Then, for a system of differential equations $K^0(x_t), K^1(x_t)$ given in the appendix, there exists an equilibrium with the following properties.

1. *Pre-development, the consumption process for EM is given by*

$$\frac{C_t^{EM}}{Y_t} = \begin{cases} K^0(x_t) & \text{if } s_t = 0 \\ K^1(x_t) & \text{if } s_t = 1 \end{cases} \tag{34}$$

and the respective consumption process for W is given by

$$\frac{C_t^{W}}{Y_t} = \begin{cases} \Omega^0(x_t) & \text{if } s_t = 0 \\ \Omega^1(x_t) & \text{if } s_t = 1 \end{cases} \tag{35}$$

where

$$\Omega^0(x_t) = \left(1 + \beta - \bar{i} f_t - K^0(x_t)\right) \tag{36}$$

$$\Omega^1(x_t) = \left(\left(1+\beta\right)\Lambda^d - \left(\bar{i} + \bar{q}\right) f_t - K^1(x_t)\right) \tag{37}$$

[7] An alternative assumption that will leave our results unaffected is that short positions in asset F_t are subject to the same monitoring costs as growth contingent contracts.

2. The equilibrium values of $p(x_t)$ and $f(x_t)$ are given as

$$p^0(x_t) = g\frac{\kappa}{\nu}\left[\frac{\beta+\kappa f^0}{\Omega^0(x_t)}\right]^{-\gamma} - \bar{i} \text{ if } s_t = 0 \tag{38}$$

$$p^1(x_t) = g\frac{\kappa}{\nu}\left[\frac{\beta+\kappa f^1}{\Omega^1(x_t)}\right]^{-\gamma} - (\bar{i}+\bar{q}) \text{ if } s_t = 1 \tag{39}$$

where the quantities $f^0(x_t), f^1(x_t)$ in the state $s_t = 0, 1$ respectively are given as the solution to the equations

$$g\frac{\kappa}{\nu}\left[\frac{\beta+\kappa f^0-\nu x_t}{\Omega^1(x_t)}\right]^{-\gamma} - \bar{i} = g\frac{\kappa}{\nu}\left[\frac{\kappa\left(1-f^0\right)+\nu x_t}{K^0(x_t)}\right]^{-\gamma}. \tag{40}$$

and

$$g\frac{\kappa}{\nu}\left[\frac{\beta+\kappa f^1-\nu x_t}{\Omega^1(x_t)}\right]^{-\gamma} - (\bar{i}+\bar{q}) = g\frac{\kappa}{\nu}\left[\frac{\kappa\left(1-f^1\right)+\nu x_t}{K^1(x_t)}\right]^{-\gamma}. \tag{41}$$

3. The equilibrium value of λ^* is given by

$$\lambda_t^* = \chi\left[p_{J=1,S=1}\overline{A}\left(\frac{\Omega^1(x_t+\widetilde{\xi}_t)}{\Omega^0(x_t)}\right)^{-\gamma} + p_{J=1,S=0}\left(\frac{\Omega^0(x_t+\widetilde{\xi}_t)}{\Omega^0(x_t)}\right)^{-\gamma}\right] \tag{42}$$

and the equilibrium value of $\widetilde{\xi}_t$ solves the equation

$$\frac{\left(\frac{K^0(x_t+\widetilde{\xi}_t)}{K^0(x_t)}\right)^{-\gamma} - \left(\frac{\Omega^0(x_t+\widetilde{\xi}_t)}{\Omega^0(x_t)}\right)^{-\gamma}}{\left(\frac{\Omega^1(x_t+\widetilde{\xi}_t)}{\Omega^0(x_t)}\right)^{-\gamma} - \left(\frac{K^1(x_t+\widetilde{\xi}_t)}{K^0(x_t)}\right)^{-\gamma}} = \frac{p_{J=1,S=1}\overline{A}}{p_{J=1,S=0}} \tag{43}$$

Proposition 2 asserts that in the absence of perfect hedging instruments it is still possible to construct an equilibrium where all the quantities depend exclusively on x_t (and s_t). In contrast to Proposition 1, the absence of a perfect hedging instrument implies that in general EM holds some reserves in its portfolio to insure against the possibility that it enters a sudden stop and the imperfect hedge does not deliver any payoffs.

For our purposes, the most illuminating equation in Proposition 2 is equation (43). This equation implies that EM *always* wants to hold some amount of imperfect hedges as long as $p_{J=1,S=1} > 0$. The easiest way to see this is to note that $\widetilde{\xi}_t = 0$ is a root of equation (43) only when $p_{J=1,S=1} = 0$. Otherwise, some amount of $\xi_t > 0$ is optimal as long as W's consumption drop upon entrance into $s_t = 1$ is smaller than EM's , i.e. as long as $\frac{\Omega^1(x_t+\widetilde{\xi}_t)}{\Omega^0(x_t)} < \frac{K^1(x_t+\widetilde{\xi}_t)}{K^0(x_t)}$.[8]

[8] To see this, note that the denominator on the left hand side of (43) is negative as long as $\frac{K^1(x_t)}{K^0(x_t)} < \frac{\Omega^1(x_t)}{\Omega^0(x_t)}$. Hence the numerator needs to be negative in order to make the left hand side and the right hand side equal. However, this

3 Estimation

Before we can proceed with a quantitative analysis of the model and its implications, we need to obtain estimates for some of the parameters of the model. In particular we are interested in the arrival and departure rates of sudden stops, and their severity. Furthermore, in order to assess the benefits of hedging, we also estimate the joint incidence of sudden stops and jumps in some appropriate financial instrument. The next section explains how we obtain these estimates.

3.1 The Sudden Stop Process

The core of our analysis is the sudden stop process. In this section we estimate the main parameters of such process: λ and $\widetilde{\lambda}$. We also estimate the magnitude of the average sudden stop reversal, $E(\theta_t^0/\theta_t^1)$, where θ_t is defined as

$$\theta_t^j \equiv 1 + f_t^j p_t^j, \; j = \{0, 1\}.$$

To simplify notation we drop the superscript from θ_t^j when there is no place for confusion. We use post 1983 data for six emerging market economies for which we had complete data and behave more or less as described by the model with no contingent instrument (see the Appendix for details on the selection criterion): Chile, Colombia, Indonesia, Malaysia, Mexico, and Thailand.

The first step is to find empirical counterparts for the processes describing available resources during NSS and SS. For this, we note that in the model these resources can be decomposed into regular income, Y_t, and financial flows, $(\theta_t - 1)Y_t$. In practice, there are several additional complexities in doing such a decomposition. These stem from the existence of multiple goods whose relative prices change during the transitions between NSS and SS and vice versa, the presence of temporary terms of trade shocks, and endogenous domestic output declines during sudden stops. The Data Appendix describes our methodology to deal with those issues. In a nutshell, we approximate Y_t with the permanent component of domestic national income, and $(\theta_t - 1)Y_t$ with the sum of capital flows in terms of imported goods and the transitory component of exports and terms of trade effects. Our main left hand side variable is the ratio of these two, which can be loosely

can only happen when $\widetilde{\xi}_t > 0$ since Ω^1 is decreasing in x_t (since W's holdings of bonds is equal to $-x_t$), whereas K^0 is increasing in x_t.

interpreted as external financing over "normal" pre-development income:

$$\psi_{it} \equiv \theta_{it} - 1 = \frac{(\theta_{it} - 1)Y_{it}}{Y_{it}},$$

where i is the country index.

Since the measurement procedure for θ_{it} can potentially produce measurement error and the data are time aggregated as opposed to continuous, we assume that ψ_{it} is observed with (state dependent) noise,

$$\widetilde{\psi}_{it} = \psi_{it} + \varepsilon_{it}(s_{it})$$

with

$$\varepsilon_{it}(s_{it}) \sim N(0, \sigma_{\varepsilon}^2(s_{it})), \qquad s_{it} \in \{0, 1\}.$$

Since ψ_{it} will exhibit jumps as the economy transits from tranquil times ($s_t = 0$) to sudden stops ($s_t = 1$) we use a standard regime-switching model a la Hamilton (1989, 1990) to estimate the average value of ψ_{it} during tranquil times (that we call ψ_i^{NSS}) and the equivalent value of ψ_{it} during sudden stops $\left(\psi_i^{SS}\right)$. As part of this procedure, we can also obtain estimates for the probability of transition from tranquil times ($s_t = 0$) to sudden stops ($s_t = 1$):[9]

$$p_i(NSS \rightarrow SS) \equiv \Pr(s_{i,t+\Delta} = 1 | s_{i,t} = 0) = 1 - e^{-\lambda_i \Delta} \tag{44}$$

$$p_i(SS \rightarrow NSS) \equiv \Pr(s_{i,t+\Delta} = 0 | s_{i,t} = 1) = 1 - e^{-\widetilde{\lambda}_i \Delta} \tag{45}$$

For the calibration exercises we convert annual transition probabilities into annual frequencies by setting:

$$\lambda = -\log\left[1 - P(NSS \rightarrow SS)\right]/\Delta \tag{46}$$

$$\widetilde{\lambda} = -\log\left[1 - P(SS \rightarrow NSS)\right]/\Delta \tag{47}$$

Given the limited number of SS observations we have for each country and the highly nonlinear nature of the hidden states model we are estimating, we use a Bayesian approach (with flat priors) based on a Gibbs Sampler (see Kim and Nelson [1999]). We describe the precise procedure in the Appendix. To obtain more precise estimates, we assume that the parameters $p_i(NSS \rightarrow SS)$, $p_i(SS \rightarrow NSS)$ are the same across all countries, whereas ψ_i^{NSS}, ψ_i^{SS} are allowed to differ. Tables

[9] These equations approximate the continuous time model by its discrete analog, by implicitly assuming that there can be at most one transition in a time interval of Δ. This approximation becomes exact as $\Delta \rightarrow 0$.

	Average	Std Dev	5%	25%	50%	75%	95%
λ	0.19	0.050141	0.11337	0.15434	0.1868	0.22153	0.27772
$\widetilde{\lambda}$	0.14769	0.046718	0.079083	0.11403	0.14322	0.17685	0.23111

Table 1: Posterior distribution of $\lambda, \widetilde{\lambda}$.

	Average	Std Dev	5%	25%	50%	75%	95%
Chile	-0.091	0.030	-0.132	-0.109	-0.094	-0.076	-0.041
Colombia	-0.051	0.009	-0.064	-0.057	-0.052	-0.047	-0.036
Mexico	-0.084	0.015	-0.106	-0.093	-0.085	-0.075	-0.059
Indonesia	-0.066	0.009	-0.081	-0.072	-0.067	-0.061	-0.049
Malaysia	-0.164	0.023	-0.199	-0.178	-0.164	-0.149	-0.126
Thailand	-0.146	0.023	-0.179	-0.161	-0.147	-0.133	-0.107

Table 2: Posterior distribution of capital flow reversal (η) during sudden stops

1 and 2 present these estimates. Based on the posterior medians, we conclude that sudden stops are large, leading to declines in available resources sometimes beyond 10 percent of GDP, their effects last for about 6.5 years and they occur about every 12 years (about 5.5 years after exiting the previous sudden stop). Our measure of sudden stop is meant to capture not only the initial spike in interest rates and turmoil but also the effects of sudden stops that remain well after the onset of the capital flow reversal. Our estimates suggest that these effects can be quite large: Countries take a long time in resuming significant borrowing after experiencing severe capital flow reversals.

Finally, Figure 2 summarizes the output of the corresponding Gibbs sampler for our economies. It illustrates the path of the share of economies with posterior probabilities of being in sudden stop above 0.5 during a given period. It is apparent from this figure that our procedure does capture the transitions into sudden stops that occur around major financial crises.

3.2 The VIX and the joint incidence of sudden stops and VIX jumps

To evaluate the usefulness of hedging, we need to find an instrument that exhibits jumps at times when a country transits into a sudden stop. If such an asset exists, then it is straightforward to use

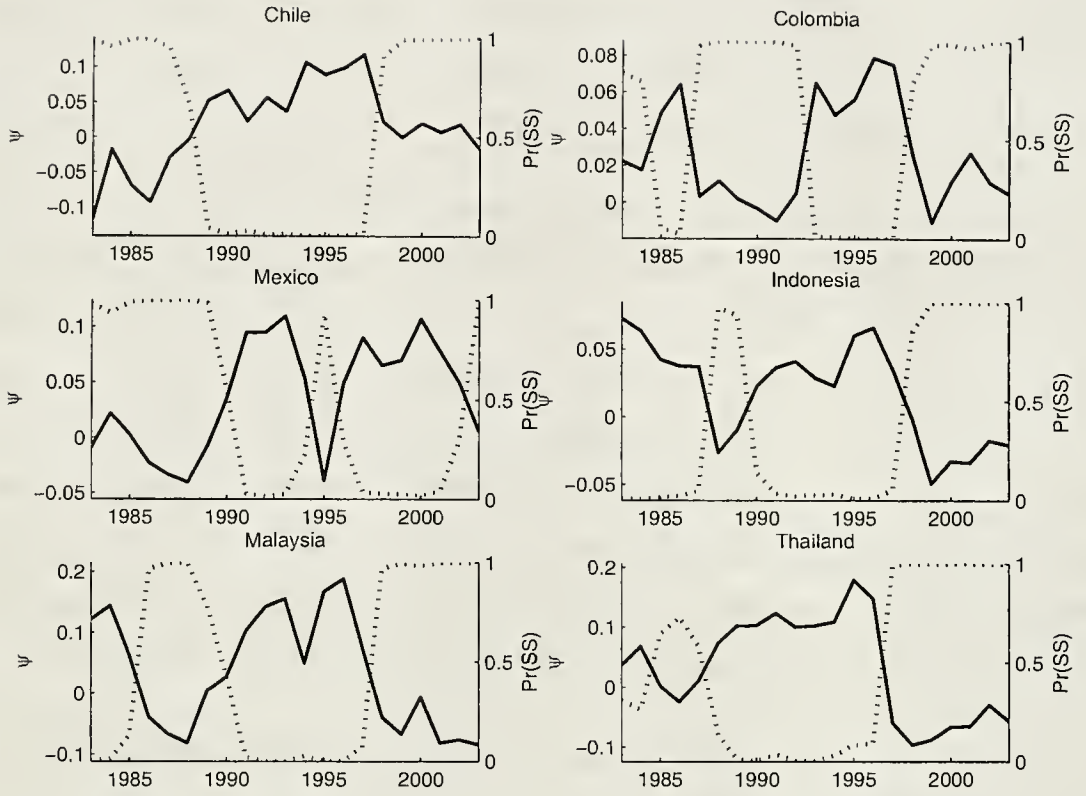

Figure 1: ψ^i and posterior probability of being in a SS for 6 different countries.

a combination of short dated put and call options on the asset in order to synthetically approximate a payoff of 1 dollar when the jump in the underlying asset occurs and 0 otherwise. (We explain in detail how to approximate such a claim with call options shortly.)

Our goal here is not to conduct a thorough search for the optimal risky instrument for specific countries' portfolios. Rather, we seek to show that there exist assets with such properties and obtain an order of magnitude of the benefits of expanding EM's strategies to include such an asset. Moreover, by finding an asset that delivers payoffs in states of the world with increased incidence of sudden stops, we can use risk premia on that asset to infer the price of obtaining payoffs in such states.

With this purpose in mind, we chose the CBOE Volatility Index (VIX). This is a traded index formed from quoted put and call options on the S&P 500, available since the mid-1980s. The VIX

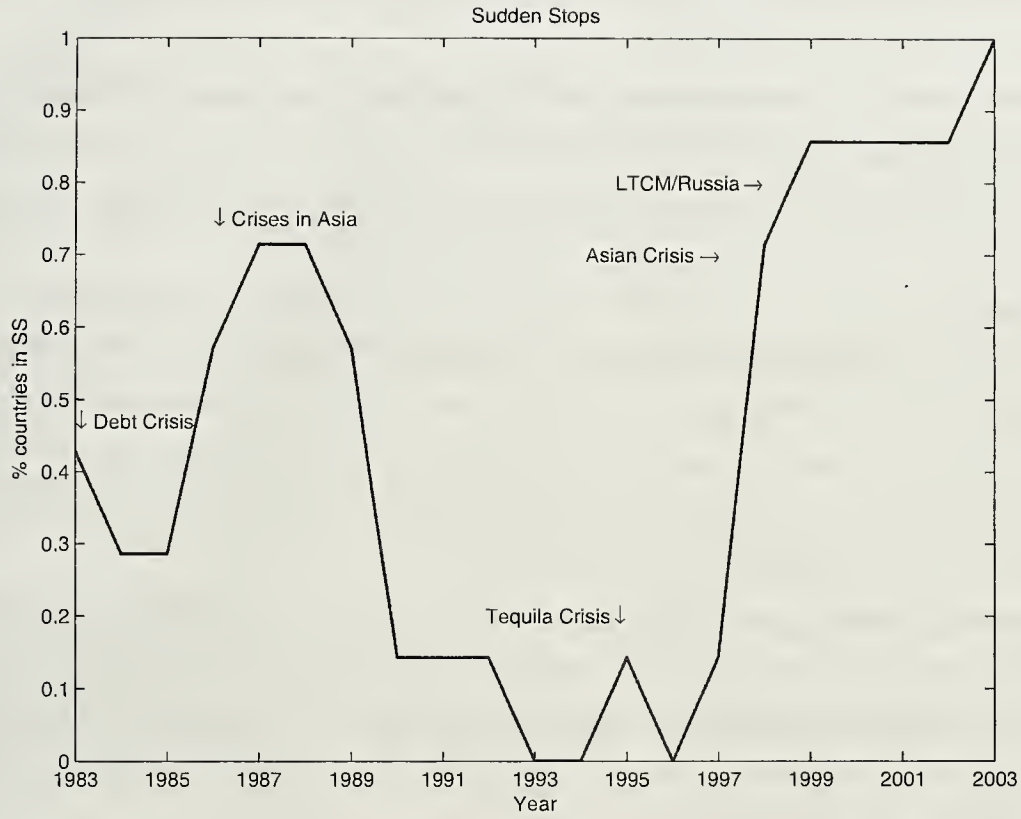

Figure 2: Fraction of Economies in SS

captures traders' anticipations of volatility in the US stock market over the next month as implied by various puts and calls on the S&P 500.[10] As we show below, sudden stops often coincide with jumps in the VIX, making it a good candidate for our purposes.[11]

[10] For more details on the construction of the VIX, see http://www.cboe.com/micro/vix/vixwhite.pdf

[11] See also the recent work of, Pan and Singleton (2007) documenting the tight correlation between EM's sovereign CDS' prices and the VIX.

3.2.1 The VIX Process

Let us postulate a continuous time process of the VIX, described by an Ornstein-Uhlenbeck (i.e. a continuous time analog of an AR(1) process) with jumps

$$d\log(VIX) = -\alpha\left(\log(VIX) - \overline{\log(VIX)}\right)dt + \sigma_{VIX}dZ_t + \left[\phi d\Phi_t - \lambda_J\mu_\phi dt\right]. \tag{48}$$

α is a parameter that controls the speed of mean reversion, $\overline{\log(VIX)}$ is the average level of (log) VIX, σ_{VIX} is the instantaneous volatility of the process, dZ_t and $d\Phi_t$ capture the increments to a standard Brownian motion and a Poisson process with intensity λ_J respectively. Finally, ϕ captures the (potentially random) magnitude of the jump in $\log(VIX)$ and is assumed to be drawn from a normal distribution with mean μ_ϕ and standard deviation σ_ϕ. Given these assumptions, the second and third terms in (48) are martingale differences.

In estimation, we approximate the above process by its discrete time counterpart. Since (48) is an Ornstein Uhlenbeck process, its discrete time observations follow an AR(1) process. Hence, the first step is to remove this predictable component by estimating an AR(1) process for $\log(VIX)$ and then isolating the residuals. For small time intervals, these residuals are characterized (approximately) by a mixture of normals:[12]

$$\varepsilon_t \overset{d}{=} (1-p_{VIX})N(\mu_{VIX}\Delta, \sigma^2_{VIX}\Delta) + p_{VIX}N(\mu_{VIX}\Delta + \mu_\phi, \sigma^2_{VIX}\Delta + \sigma^2_\phi) \tag{49}$$

where $p_{VIX} = 1 - e^{-\lambda_J\Delta}$, $\mu_{VIX} = -\lambda_J\mu_\phi\Delta$.

In principle, estimation of this process can proceed from this point on along standard estimation techniques (see, e.g., Caballero and Panageas [2004]). Such estimation delivers estimates of the underlying parameters of the process for $\log(VIX)$. Given these parameters, we can also estimate the posterior probabilities that the VIX has exhibited a jump in a given month, and we can then also estimate the joint incidence of jumps in the VIX and the arrival of sudden stops.

Unfortunately, these estimates are not directly useful in practice, because in real markets one can only find contracts whose payoffs are contingent on the level of the VIX (say, at the end of the month). However, there are no contracts on whether the VIX has exhibited a jump over the

[12] The source of the approximation to the continuous time limit is that the discrete approximation excludes the possibility of more than one jump in the interval Δ, which seems reasonable if we want to focus on relatively large and infrequent jumps and relatively small time intervals Δ.

course of a month, because such jumps are unobservable (and impossible to define) with discrete time trades.

To measure the correlation between the arrival of sudden stops and the payoffs of a feasible strategy, we adopt an alternative procedure.[13] In particular, we first perform a Bayesian estimation of (49) using monthly data. We then use the obtained estimates and set a cutoff value $\overline{e}$, such that

$$Pr(\varepsilon_t > \overline{e}|J = 0) < 0.01.$$

In a next step we let the expected level of $\log(VIX)$ after a time interval of Δ be defined as

$$\log(VIX^e_{t+\Delta}) \equiv (1 - \alpha\Delta)\log(VIX_t) - \alpha\Delta\overline{\log(VIX)}$$

and consider a contract that delivers the following payoffs:

$$Q_{t+\Delta} = \begin{cases} 1 \text{ if } \log(VIX_{t+\Delta}) > \log(VIX^e_{t+\Delta}) + \overline{e} \\ 0 \qquad\qquad\qquad\qquad \text{otherwise} \end{cases} \tag{50}$$

That is, this contract delivers a payoff of 1 if the unexpected change in the $\log(VIX)$ over an interval of Δ is above $\overline{e}$. It is also important to note that in the limit where Δ becomes arbitrarily small, this contract pays off 1 if and only if there is a jump in the VIX and 0 otherwise. Hence, for small Δ, the payoff $Q_{t+\Delta}$ approaches the contract described in section 2.5.1.[14] Given that (50) is a payoff that is measurable with respect to the market participants' information set at time $t + \Delta$, it is more useful from this point to think directly of the event $Q_{t+\Delta} = 1$ as a "jump".

Furthermore, figure 3 illustrates how to approximate the payoff $Q_{t+\Delta}$ by combining a standard call option with strike price $\log(VIX^e_{t+\Delta}) + \overline{e}$ together with a short call option with strike price $\log(VIX^e_{t+\Delta}) + \overline{e} + \delta$. The resulting payoff is[15]

$$\widehat{Q}_{t+\Delta} = \begin{cases} \delta \text{ if } \log(VIX_{t+\Delta}) > \log(VIX^e_{t+\Delta}) + \overline{e} + \delta \\ 0 \text{ if } \qquad \log(VIX_{t+\Delta}) < \log(VIX^e_{t+\Delta}) + \overline{e} \end{cases} \tag{51}$$

As was shown by Breeden and Litzenberger (1978), when $\delta \to 0$ we obtain that $lim_{\delta\to 0}\frac{\widehat{Q}_{t+\Delta}}{\delta} = Q_{t+\Delta}$ a.e. Given this approximation result, and the existence of a rich set of options of various

[13] If anything, this procedure biases the results toward finding a lower "correlation", because it introduces the potential of "over-identifying" jumps, and hence understating the benefits of hedging.

[14] When $\Delta = 1/12$ (i.e. monthly data) the probability that $Q_{t+\Delta}$ will be 1 when no jump has occured over the course of that month is 1%. This is so by the construction of $\overline{e}$.

[15] When $\log(VIX_{t+\Delta}) \in [\log(VIX^e_{t+\Delta}) + \overline{e}, \log(VIX^e_{t+\Delta}) + \overline{e} + \delta]$ then $\widehat{Q}_{t+\Delta} = \log(VIX_{t+\Delta}) - (\log(VIX^e_{t+\Delta}) + \overline{e})$.

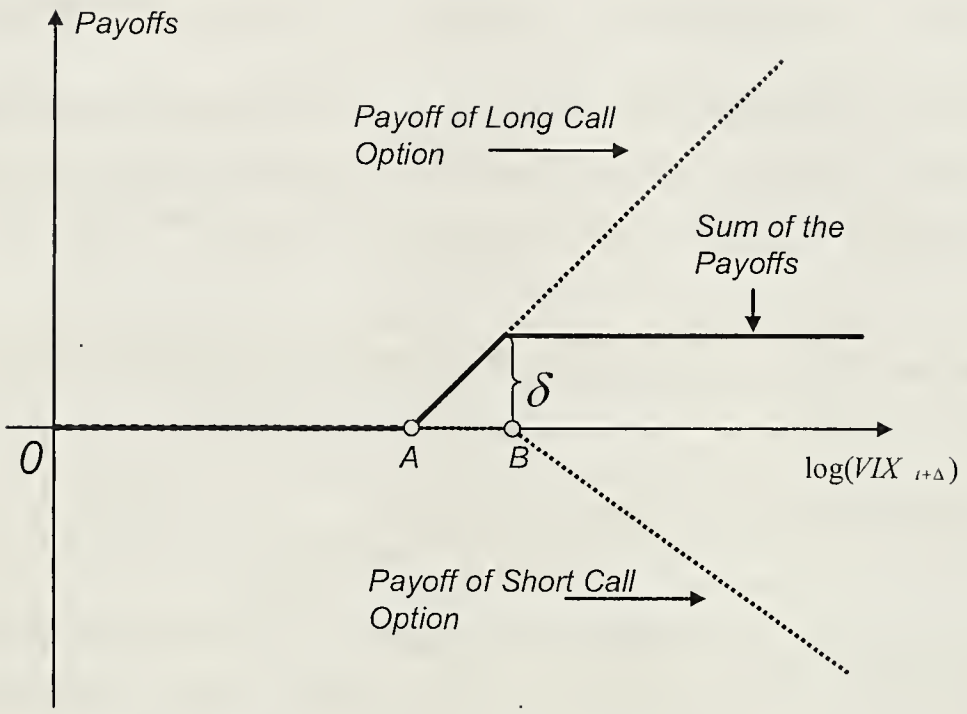

Figure 3: Payoffs of a long call option with strike price $A = \log(VIX^e_{t+\Delta}) + \overline{e}$ and a short call option with strike price $B = \log(VIX^e_{t+\Delta}) + \overline{e} + \delta$. The solid line presents the sum of the two payoffs.

strike prices, we assume directly that the payoff given by (50) is a feasible payoff given existing securities.

Importantly for our purposes, equation (51) implies that the arbitrage free price of the payoff $Q_{t+\Delta}$ for small δ is given (approximately) as the price difference of two standard call options with strike prices $\log(VIX^e_{t+\Delta}) + \overline{e}$ and $\log(VIX^e_{t+\Delta}) + \overline{e} + \delta$ respectively, divided by δ. Denoting that price as $P^{Q_{t+\Delta}}$, a feasible strategy to replicate a payoff such as (29) in section 2.5.1 is given as follows: Every time interval Δ (say every one or two months) an EM can pay $P^{Q_{t+\Delta}}$ and obtain a payoff such as (50). If the (unexpected) change in the VIX is less than $\overline{e}$, then $Q_{t+\Delta} = 0$. If however, there is a discontinuity that produces an (unexpected) change larger than $\overline{e}$, then $Q_{t+\Delta} = 1$. Hence, the risk compensation λ^* in equation (29) as[16] $\lambda^* \simeq P^{Q_{t+\Delta}}/\Delta$. In the appendix we give further details on how we used the prices of existing VIX call options to obtain an estimate for the average

[16] In the limit where $\delta \to 0$ and $\Delta \to 0$ this approximation becomes exact.

	Average	Std Dev	5%	25%	50%	75%	95%
$Pr(SS\|J)$	0.236	0.096	0.095	0.165	0.226	0.296	0.413
$Pr(J)$	0.159	0.032	0.109	0.137	0.158	0.180	0.215

Table 3: Posterior distribution of $Pr\left(SS|J\right)$ and $\Pr(J)$ based on quarterly data.

value of $P^{Q_{t+\Delta}}$ and hence λ^*. Based on that analysis our baseline value for λ^* is 0.94.

3.2.2 Joint jumps, risk premia and implied preference shocks

It is now straightforward to measure the joint incidence of sudden stops and positive payoffs to a claim such as the one defined in equation (50). Figure 4 gives a first visual depiction of this joint incidence. It shows the residuals of an AR(1) model for $\log(VIX)$. The shaded areas represents those instances when these residuals are above $\overline{e}$ and hence the times when a claim like the one constructed in equation (50) would pay off 1 dollar. The shaded areas occur in the early 90's (at the onset of the gulf war) in 1997 (around the Asian crisis), in 1998 (around the Russian/LTCM crisis), after 9/11/2001, and around the beginning of the U.S. corporate scandals and the Argentinean default.

These large movements in the VIX at times of crisis, together with the increased incidence of sudden stops during such times, suggest that there is some joint incidence of the two events, which can be used for hedging purposes. We will refer to the times when the payoff of equation (50) is 1 as "jump" times and we will denote such events with $J = 1$ in analogy to section 2.5.1.

Conditioning on the times where $J = 1$, it is straightforward to use a Bayesian procedure similar to section 3.1 in order to estimate the parameters

$$P(SS|J=1) \equiv \frac{p_{J=1,s=1}}{p_{J=1,s=0} + p_{J=1,s=1}} \text{ and } P(J) = 1 - e^{-\lambda_J \Delta}$$

where $p_{J=1,s=1}, p_{J=1,s=0}$ etc. were defined in section 2.5.1. The exact procedure is described in the appendix. The resulting posterior distributions are given in table 3.

With the estimates of $P(J)$ from table 3 we can obtain estimates of the arrival intensity λ_J in a manner analogous to equations (46) and (47) , namely $\lambda_J = -\log\left[1 - P(J)\right]/\Delta$. In turn, given the estimates for λ (from section 3.1) and also λ_J, and $P(SS|J=1)$ from table 3, we can identify

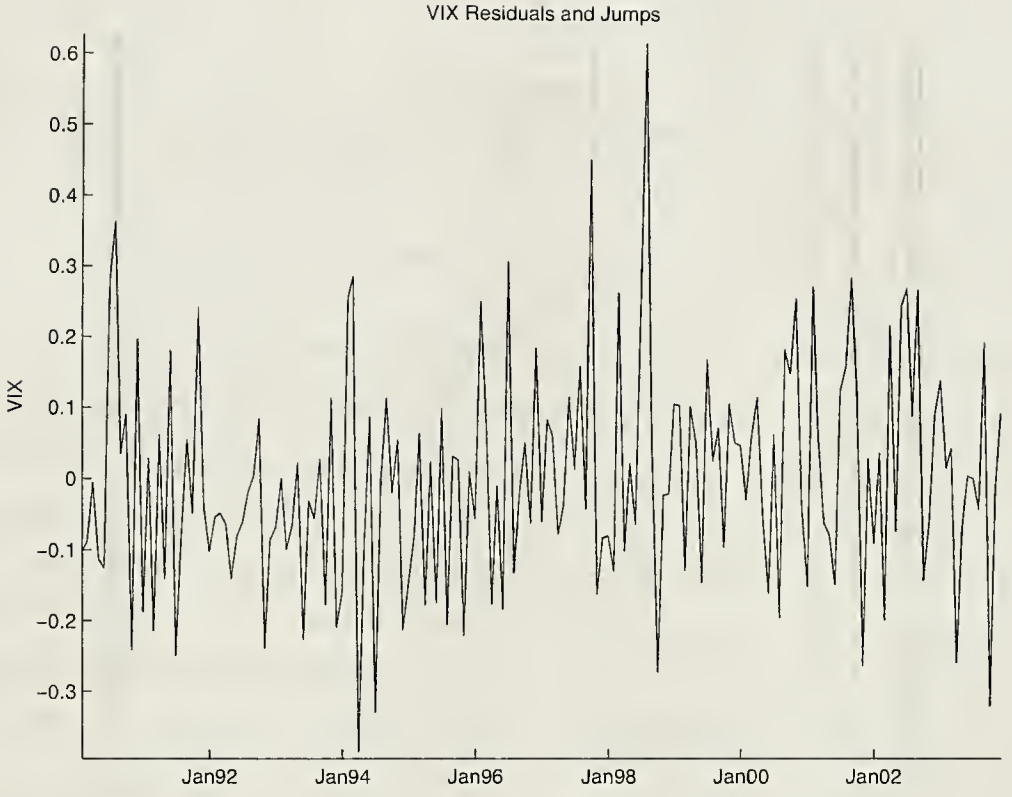

Figure 4: VIX residuals. The grey areas correspond to instances where the VIX residual is above the jump-cutoff.

all the parameters of section 2.5.1 by solving the following linear system of equations

$$\lambda_J \equiv \chi\left(p_{J=1,s=1} + p_{J=1,s=0}\right) \tag{52}$$

$$\lambda \equiv \chi\left(p_{J=1,s=1} + p_{J=0,s=1}\right) \tag{53}$$

$$\Pr(SS|J) = \frac{p_{J=1,s=1}}{p_{J=1,s=0} + p_{J=1,s=1}} \tag{54}$$

$$1 = p_{J=1,s=1} + p_{J=1,s=0} + p_{s=1,J=0} \tag{55}$$

for the four unknowns $p_{J=1,s=1}$, $p_{s=1,J=0}$, $p_{J=1,s=0}$ and χ. Based on the median posterior values for λ, λ_J, and $P(SS|J=1)$, the values of $p_{J=1,s=1}$, $p_{s=1,J=0}$, $p_{J=1,s=0}$ and χ that solve the system of equations (52)-(55) are $\chi = 0.74$, $p_{J=1,s=1} = 0.23$, $p_{s=1,J=0} = 0.06$, $p_{s=1,J=0} = 0.71$.

4 Calibration and Results

In this section we input the estimates obtained in the previous section into a calibration exercise, and then use the resulting quantitative model to evaluate the gains from different precautionary strategies. Since the standard practice of central banks in emerging markets is to hold reserves almost exclusively in the form of noncontingent assets, we calibrate the model of section 2.5.1 assuming that a country does not use any hedging instruments. This standard practice seems to be more the result of inertia and domestic political economy factors, and hence it is reasonable to ask how much would welfare increase if countries adopted optimal hedging strategies. All the counterfactual exercises are done in general equilibrium, and hence take into account the endogenous reaction of prices.

4.1 Parameters

We calibrated the model following two different approaches. In the first approach we use standard CRRA preferences with no global preference shock ($A = 1$ for all t). As with all calibrations of this kind, this approach is able to match quantity data well but not the large risk premia observed in actual markets, especially during times of turmoil. Within this first approach, we allow for two subcases -one where cyclical components in world output are small (case 1a) and hence sudden stops are caused almost exclusively by changes in monitoring costs, and one where sudden stops coincide with significant global downturns (case 1b). In the second approach (case 2) we allow for jumps in the preference shock A_s in order to calibrate interest rates and fluctuations in the global price of risk. We also allow in this case for a drop in growth rates during sudden stops, which helps us fit the volatility of interest rates. The main difference between these cases is that W's willingness to insure EM varies across them.

Table 4 reports the parameters for the three cases we consider. The first twelve parameters are common across all cases. The parameter g is set to 0.03 to approximately match the speed of convergence estimated by Barro and Sala-i-Martin (2003). We set $\kappa = 3$, so that the expected rate of (logarithmic) growth of income in an emerging market economy $g \log(\kappa) = 3.2\%$ higher than that of a developed economy. The parameter β controls the relative size of EM and W in the model. Since we are interested in estimating the benefits of hedging for a large part of the world, such

	Case 1a	Case 1b	Case 2
g	0.03	0.03	0.03
κ	3	3	3
β	7	7	7
$\widetilde{\lambda}$	0.14	0.14	0.14
χ	0.74	0.74	0.74
$P_{S=1,J=1}$	0.23	0.23	0.23
$P_{S=0,J=1}$	0.71	0.71	0.71
$P_{S=1,J=0}$	0.06	0.06	0.06
$\overline{i}$	0.01	0.01	0.01
$\overline{q}$	0.35	0.35	0.35
γ	7	7	7
ρ	0.01	0.01	0.01
σ	0.027	0.02	0.02
μ_0	0.018	0.018	0.03
μ_1	0.018	0.018	0
μ_G	0.018	0.018	0.018
$\overline{A}$	1	1	1.8
Λ^d	0.99	0.96	0.96

Table 4: Parameters used in the calibration exercises.

as Latin America, we added the (PPP adjusted) GDP of US, EU countries and Japan and then compared that to the GDP of Latin American countries.[17] The parameters $p_{J=1,s=1}$, $p_{s=1,J=0}$, $p_{J=1,s=0}$ and χ are those that solve the system of equations (52)-(55) as determined in the previous section. $\widetilde{\lambda}$ was estimated and reported in table 1. We chose $\bar{i}$ so as to match capital flows of about 9% of GDP in tranquil times. The value of $\bar{q}$ is set high enough, so that in all three cases capital flows become practically zero during sudden stops. We set ρ to a low number in order to reduce the high risk-free rates that are typically obtained, when one uses CRRA preferences. Finally, we chose γ to generate reserves accumulation in the $20-30\%$ range.

The next set of parameters varies across the different cases. For cases 1a and 1b we set the growth rates of Y to $\mu_0 = \mu_1 = \mu_G = 0.018$, so as to capture growth rates in developed economies (see Campbell and Cochrane 1999). In case 2 we preserve (approximately) the average growth rate but assume some variation in the growth rate of Y in order to match the observed volatility in real riskless interest rates.[18] In case 2 we calibrate A to approximately match the observed VIX's risk premia (assuming that originally only W participates in this market). Finally, in all cases we set Λ_t to capture the standard deviation of the cyclical components in output in the US. The drop in case 1a produces a standard deviation of the cyclical component corresponding to what is obtained from a Beveridge-Nelson decomposition of US output.[19] The drop in cases 1b and 2 produces a model-implied standard deviation of cyclical component that corresponds to the one estimated from a standard H-P filter decomposition for the US.[20] We set σ_Y so that in all three cases the annual volatility of output produced by the model is about 0.028, corresponding approximately to annual post-war US data.[21]

[17] We also estimated the model by matching the relative size of US, EU and Japan to Latin American countries and East Asian economies (Indonesia, Korea, Singapore, Thailand and Malaysia) obtaining similar results.

[18] The presence of preference shocks necessarily increases the volatility of the interest rate. By allowing for some small time variation in output growth that volatility can be reduced within reasonable levels. Since in case 2 we are interested in matching asset prices, we allowed such time variation in the stochastic trend. However, we also simulated the model allowing for constant growth rates and preference shocks. This did not affect the results substantively.

[19] See Nelson (2006), p. 12.

[20] See Nelson (2006), p. 12.

[21] Note that the total variance of output is given as $E\left[Var_t\left(Y_{t+1}-Y_t\right)\right]+Var\left[E_t\left(Y_{t+1}-Y_t\right)\right]$. Since in case 1a) the variance of the predictable component $Var\left[E_t\left(Y_{t+1}-Y_t\right)\right]$ is smaller, we assign a larger value to σ_Y, so that total volatility stays the same compared to case 1b. In case 2, the variation in growth rates μ_0, μ_1 tends to reduce somewhat the total volatility of output compared to case 1*b*, but this effect has a negligible effect on total volatility

	Model			Data
	Case 1a	Case 1b	Case 2	
Mean Reserves	0.31	0.32	0.2	0.17
Mean Resource Drop	0.091	0.091	0.091	0.1
Jump Risk Intensity ratio (λ^*/λ_J)	1.01	1.04	1.33	1.39
Mean Interest Rate	0.14	0.14	0.01	0.020
Volatility of the interest rate	0.008	0.04	0.04	[0-0.05]

Table 5: Comparison between model and data, assuming no hedging.

Table 5 reports several statistics generated by the model when only noncontingent reserves are accumulated and the corresponding values in our sample of countries.[22] The numbers in the data column are based on the following: Reserves refers to the historical average levels of reserves for the countries that we consider as reported in Table 9 in the appendix. Mean resource drop refers to the average reversal in capital flows (as a fraction of GDP) during the sudden stop as reported in table 2. The jump risk intensity ratio is obtained by our estimates of λ_J and λ^* in the previous section as $\frac{\lambda^*}{\lambda_J}$ (assuming that only W participates in this market). The data on the average real interest rate are from Campbell and Cochrane (1999), while the volatility of the real interest rate is from (Jermann 1998). (In the data one can only obtain estimates about the volatility of the realized real return on bonds. These estimates provide an upper bound on the volatility of the unobserved *anticipated* real interest rate, and hence we report the volatility of the *anticipated* real interest rate as an interval.) The models produce reasonable numbers for reserve accumulation and the size of the sudden stops. However, as explained above, only case 2 is designed to match asset prices by introducing an additional source of shocks to the marginal utility of consumption. In particular, the jump in A_s is needed to generate jumps in marginal utility consistent with the rise in the "risk premium" implicit in the VIX during crises. Jumps in A_s also help reduce the interest rate, since they generate a positive drift in the marginal utility of consumption.

(about 20 basis points reduction in annual volatility).

[22] To obtain the stationary quantities we simulated 20000 artificial yearly data, conditioning on no transition to development.

4.2 Hedging

	Case 1a	Case 1b	Case 2
Portfolio (imperfect correlation)	0.87	0.86	0.78
Portfolio (perfect correlation)	1.00	1.00	1.00
Proportional reduction in stationary reserves (imperfect correlation)	0.16	0.16	0.23
Proportional reduction in stationary reserves (perfect correlation)	0.42	0.47	0.62

Table 6: Portfolio allocated to hedging and proportional reduction in stationary non-contingent reserves (reserves prior to hedging divided by reserves after hedging).

In this section we study the effect of hedging on portfolio decisions and welfare. Table 6 focuses on the former. The first two rows report the share of the flow of precautionary savings spent in hedging, that is $\lambda^* \xi_t / (r_t X_t + \theta_t Y_t - C_t)$, for the case where only options on VIX are available (imperfect correlation) and for an upper bound where an asset perfectly correlated with the occurrence of a sudden stop is available (perfect correlation). We refer to this ratio as the "portfolio" invested in hedging instruments. The portfolios are evaluated at $x_t = 0$.[23] The message of these two rows is clear in all cases: the bulk of the precautionary resources, even when the correlation is imperfect, should be allocated to hedging instruments rather than reserves accumulation.

We already knew from Proposition 1 that this "portfolio" notion is always 1 when there exists sudden stop insurance that is both perfectly correlated with the sudden stop and also insures EM against the duration of the sudden stop ("complete risk sharing"). Once the price of risk is correctly assessed, a country always prefers a hedging instrument to non-contingent reserves, irrespective of the magnitude of preference shocks A and the associated risk premia. Table 6 shows that this mechanism remains intact even when there is no insurance against the duration of sudden stops (second row). This mechanism is weakened but still remains dominant (first row) even when hedging is limited to VIX instruments.

The third and fourth rows of 6 describe a stock rather than a flow concept. They are based on

[23] Evaluating the portfolio at $x_t = 0$ is conservative. When we solve the model for all three cases and we compute the portfolio at levels of x_t that are higher than zero, the denominator $r_t X_t + \theta_t Y_t - C_t$ decreases faster than the numerator $\lambda^* \widetilde{\xi}_t$, making the portfolio rise toward 1.

a simulated run of the model with and without hedging.[24] The two rows report the proportional decline in average non-contingent reserves that is achieved with various forms of hedging in the different cases that we consider. The table shows that EM holds between 16 and 62 percent less reserves when it can hedge.

What is the impact of these changes on welfare? We assess the gains from hedging strategies by computing the equivalent variations in income required to compensate for the absence of a particular form of insurance. For this, let us define the no-insurance case as one in which EM only trades in growth contingent contracts without any access to either noncontingent bonds or any form of hedging. In such a world EM's utility is:

$$V^{no\_ins}(Y_0) = \mathrm{E}_0 \int_0^{\tau^G} A_s \frac{(\Lambda_s + f_s p_s)^{1-\gamma} Y_s^{1-\gamma}}{1-\gamma} e^{-\rho(s-t)}\, ds + V^{dev}$$

where

$$V^{dev} \equiv \mathrm{E}_0 \int_{\tau^G}^{\infty} A_s \frac{\kappa^{1-\gamma}(1-f_{\tau^G})^{1-\gamma} Y_s^{1-\gamma}}{1-\gamma} e^{-\rho(s-\tau^G)}\, ds$$

Correspondingly, let $V^{res}(0, Y_0)$ and $V^{ins}(0, Y_0)$ denote the EM's utility in a world in which the country is allowed to use reserves only to insure against sudden stops and hedging instrument *ins* (in addition to reserves), respectively, both evaluated at zero initial reserves. Then, the income variation is defined as the additional pre-development income that needs to be given to an EM with no access to insurance or reserves, in order to achieve $V^{res}(0, Y_0)$ and $V^{ins}(0, Y_0)$ respectively. Since utility is homogeneous of degree $1-\gamma$ with respect to the country's income, we have that:

$$\begin{aligned} V^{res}(0, Y_0) &= (1+k^{res})^{1-\gamma}\, \mathrm{E}_0 \int_0^{\tau^G} A_s \frac{(\Lambda_s + f_s P_s)^{1-\gamma} Y_s^{1-\gamma}}{1-\gamma} e^{-\rho s}\, ds + V^{dev} \\ V^{ins}(0, Y_0) &= (1+k^{ins})^{1-\gamma}\, \mathrm{E}_0 \int_0^{\tau^G} A_s \frac{(\Lambda_s + f_s P_s)^{1-\gamma} Y_s^{1-\gamma}}{1-\gamma} e^{-\rho s}\, ds + V^{dev} \end{aligned}$$

where the k's represent the proportional income variation. Since our goal in this section is to evaluate the improvement over the standard practice of accumulating noncontingent reserves, we

[24] See footnote 22 for a description of the simulation design. Importantly, when we compute average reserves, we also take into account states of the world when $s_t = 1$. When EM has no way to insure the duration of the sudden stop (as in Proposition 1), it has to hold non-contingent bonds in this regime. Because of this, even when there is perfect correlation between the arrival of the sudden stop and the payoffs of the contingent instrument, the stationary reserves will always be non-zero, unlike in Proposition 1.

report the welfare gains as the proportional gain over k^{res}.[25] Table 7 shows that the extra benefits

	Case 1a	Case 1b	Case 2
Imperfect Correlation	20	15	8
Perfect Correlation	91	60	44
Complete Markets	114	81	60

Table 7: Percent increase in the relative effectiveness of hedging $\left(k^{ins} - k^{res}\right)/k^{res}$, for various cases.

from enriching the precautionary strategy with hedging instruments ranges between 8 and 20% of k^{res} when only VIX is used, from 44 to 91% in the perfect correlation scenario, and from 60 to 114% as we move to the complete markets case. Importantly, all these are general equilibrium results and therefore do take into consideration the endogenous changes in the price of risk. The reason for the difference between the three cases is that in cases 1b and 2 the transition into state $s_t = 1$ "hurts" W more than in case 1a. (In case 1b W's income falls and in case 2 W's marginal utility increases). This means that the potential for risk sharing is larger in case 1a than in the others, making the benefits of hedging larger.

5 Final Remarks

Emerging market economies hold levels of international reserves that greatly exceed the levels held by developed economies (relative to their size). This would seem paradoxical given that, unlike the latter, the former face significant financial constraints with much of their growth ahead of them. The paradox disappears once these greater financial constraints also become an important source of volatility, which countries seek to smooth. This is the context we have modelled, analyzed, and assessed quantitatively.

[25] The absolute value of k^{res} itself is rather small in line with all welfare calculations in the Lucas tradition. For instance in cases 1a), 1b) and 2) k^{res} is 0.26, 0.32 and 0.23 percent of GDP respectively. This is in the same order of magnitude as Lucas's estimates of the costs of business cycles (0.1 percent of GDP). Since the absence of heterogeneity, and any links between temporary and permanent consumption components may be understating the magnitude of welfare gains, we use a concept of proportional increases in welfare benefits that should be more immune to these problems. An added advantage is that it makes our results less sensitive to assumptions about risk aversion etc.

The main contributions of this paper are twofold: First, we develop a quantitative global-equilibrium model of sudden stops. Second, we use this structure to discuss practical mechanisms to insure emerging markets against sudden stops, ranging from conventional non-contingent reserves accumulation to more sophisticated contingent strategies. Depending on the source of sudden stops, their correlation with world events, and the quality of the hedging instrument available, the gains from these strategies can be substantial.

On the first contribution, the model is successful in matching the extent of capital flow reversals, the behavior of risk premia and the size of reserves accumulation. On the second one, we estimate that the potential gains from adopting hedging strategies are, in a worst case scenario, about 10 percent more efficient than conventional reserves management, but the gains can also reach up to 115 percent as the quality of hedging instruments improves and the correlation between sudden stop arrivals and marginal investors' risk attenuates. All these strategies imply a substantial reduction in the rate of reserves accumulation and in many practical instances may free up reserves for productive investments. For example, in our case 2 (which calibrates quantities and prices), we compute that our representative country would need to hold between 23-62% less reserves than it does without hedging (depending on the quality of the hedging instrument).

There are several natural extensions to our work. Our model assumes preference shocks and frictionless trading in (imperfect) hedging instruments to explain observed risk premia. This mechanism allows us to reproduce variations in the marginal utility of consumption of the representative agent in a spirit similar to Campbell and Cochrane (1999). As a first pass, this approach is useful since it safeguards that welfare computations are consistent with asset prices. An interesting extension of our model would be to assume some form of segmentation between bond markets and hedging instruments and derive the observed risk premia from the heterogenous participants in these markets. It is reasonable to conjecture that in such a world, increasing participation in the markets for (imperfect) hedging instruments by agents in the developed world would make the outcomes resemble the cases 1a) and 1b) in our model.

Furthermore, we have assumed a representative agent within each of the regions of the world. It is quite apparent that in reality there are plenty of financial frictions within each of these regions that clearly interact with the aggregate risk management and sharing problem we have discussed. By the same token, it seems important to endogenize the production side of the economy, as in

practice real investment and employment are severely affected by sudden stops.

6 Appendix

A Proofs

Proof of Proposition 1. We start by focusing first on development. By the results in (Karatzas and Shreve 1998), Chapter 4, the market post development is dynamically complete and hence there exists a stochastic discount factor H_t such that

$$e^{-\rho(t-\tau_+^G)}\left(\frac{C_t^W}{C_{\tau_+^G}^W}\right)^{-\gamma} = e^{-\rho(t-\tau_+^G)}\left(\frac{C_t^{EM}}{C_{\tau_+^G}^{EM}}\right)^{-\gamma} = \frac{H_t}{H_{\tau_+^G}} \text{ for all } t \geq \tau_+^G \tag{56}$$

The value of a claim to W's endowment post development for any $t > \tau^G$ is then

$$P_t^W = \beta E_t \int_t^\infty \frac{H_s}{H_t} Y_s ds. \tag{57}$$

The absence of arbitrage implies that the value of the payouts that W is collecting from EM post development is $\frac{\kappa}{\beta} f_{\tau^G} P_{\tau_+^G}^W$, so that W's intertemporal budget constraint post-development is

$$\left(1 + \frac{\kappa}{\beta} f_{\tau^G}\right) P_{\tau_+^G}^W + X_{\tau^G} = E_{\tau_+^G} \int_{\tau_+^G}^\infty \frac{H_t}{H_{\tau_+^G}} C_t^W dt \tag{58}$$

Using (56) inside (58) and simplifying, leads to

$$\left(1 + \frac{\kappa}{\beta} f_{\tau^G}\right) P_{\tau_+^G}^W + X_{\tau^G}^W = C_{\tau_+^G}^W \left[E_{\tau_+^G} \int_{\tau_+^G}^\infty \frac{H_t}{H_{\tau_+^G}} \frac{C_t^W}{C_{\tau_+^G}^W} dt\right] = C_{\tau_+^G}^W \left[E_{\tau_+^G} \int_{\tau_+^G}^\infty e^{-\rho(t-\tau_+^G)} \left(\frac{C_t^W}{C_{\tau_+^G}^W}\right)^{1-\gamma} dt\right] \tag{59}$$

The market clearing condition (12) together with (56) implies that

$$\frac{H_t}{H_{\tau_+^G}} = e^{-\rho(t-\tau_+^G)} \left(\frac{Y_t}{Y_{\tau_+^G}}\right)^{-\gamma} \text{ for all } t \geq \tau_+^G \tag{60}$$

Using (60) inside (59) and (57) allows us to compute the expectation of the integral on the right hand side of (59) explicitly. Using the fact that Y_t is lognormally distributed, allows us to compute W's consumption at time τ_+^G as

$$\frac{C_{\tau_+^G}^W}{Y_{\tau_+^G}} = (\beta + \kappa f_{\tau^G}) + \nu x_{\tau^G}^W. \tag{61}$$

For future reference, it will also be convenient to combine (61) with (56) and (1) to obtain the value function of W at time τ_+^G as $A^{N_{\tau^G}} V^G(X_{\tau^G}, Y_{\tau^G})$ where

$$V^G(X_{\tau^G}, Y_{\tau^G}) = \frac{Y_{\tau^G}^{1-\gamma}}{1-\gamma} \frac{1}{\nu} \left[(\beta + \kappa f_{\tau^G}) + \nu x_{\tau^G}^W\right]^{1-\gamma}. \tag{62}$$

The rest of the proof is devoted to showing that the prices and quantities of proposition 1 constitute an equilibrium. We shall verify this directly. Verifying that markets clear is straightforward and we omit it to save space. It remains to check if the processes for $C_t^W, C_t^{EM}, i_t, f_t, n_t$ are optimal. We start by showing optimality for W. Since the preference shocks enter multiplicatively in the objective, the value function is homogenous in $\overline{A}^{N_t}$ and hence W's value function can be expressed as $\overline{A}^{N_t} V^0(X_t, Y_t)$ when $s_t = 0$ and as $\overline{A}^{N_t+1} V^1(X_t, Y_t)$ when $s_t = 1$. Specifically, the Hamilton Jacobi Bellman (HJB) equation for W when $s_t = 0$ is given by

$$\begin{aligned} 0 &= \max_{C_t^W, f_t, n_t} \left(\frac{\left(C_t^W\right)^{1-\gamma}}{1-\gamma} + V_X^0 \left(r_t X_t^W - C_t^W + \left(\beta - f_t\left(p_t + \overline{i}\right) + n_t \pi_t\right) Y_t \right) + g V^G\left(X_t^W, Y_t; f_t\right) + \lambda \overline{A} V^1\left(X_t^W, Y_t; n_t\right) \right) \\ &\quad + V_Y^0 Y \mu_0 + \frac{1}{2} V_{YY}^0 \sigma_Y^2 Y^2 - (\rho + \lambda + g) V^0 \end{aligned} \tag{63}$$

where V^G is the value function in development. Similarly, the Hamilton Jacobi Bellman equation for W when $s_t = 1$ is given by

$$\begin{aligned} 0 &= \max_{C_t^W, f_t} \left(\frac{\left(C_t^W\right)^{1-\gamma}}{1-\gamma} + V_X^1 \left(r_t X_t^W - C_t^W + \left(\beta \Lambda^d - f_t\left(p_t + \overline{i} + \overline{q}\right) - n_\tau\right) Y_t \right) + g V^G\left(X_t^W, Y_t; f_t\right) + \widetilde{\lambda} V^0\left(X_t^W, Y_t\right) \right) \\ &\quad + V_Y^1 Y \mu_0 + \frac{1}{2} V_{YY}^1 \sigma_Y^2 Y^2 - \left(\rho + \widetilde{\lambda} + g\right) V^1 \end{aligned} \tag{64}$$

Equations (64) and (63) imply the following first order conditions for C_t^W

$$\left(C_t^W\right)^{-\gamma} = V_X^j, \text{ for } j = 0, 1 \tag{65}$$

Similarly the first order conditions for f_t in the states $j = 0, 1$ can be written as

$$p_t^j = g \frac{V_f^G}{V_X^j Y_t} - \left(\overline{i} + j\overline{q}\right) = g \frac{\kappa}{\nu} \frac{\left(\beta + \kappa f_t^j\right)^{-\gamma}}{\left(c^{j,W}\right)^{-\gamma}} - \left(\overline{i} + j\overline{q}\right) \text{ for } j = 0, 1 \tag{66}$$

where ν is given by (16). The first equation in (66) follows by re-arranging the first order condition for f_t and the second equality follows by combining (65) with (62) and noting that in the postulated allocation $X_t^{EM} = X_t^W = 0$ for all t. Note that equation (66) is identical to the equations (19) and (21). Repeating the same steps as in equations (63), (64), (65) and (66) for EM gives

$$p_t^j = g \frac{\kappa^{1-\gamma}}{\nu} \frac{\left(1 - f_t^j\right)^{-\gamma}}{\left(c^{j,EM}\right)^{-\gamma}} \text{ for } j = 0, 1 \tag{67}$$

To determine the optimal choice of n_t for W in state 0, we shall first compute $V_n^1\left(X_t^W, Y_t; n_t\right)$. Letting τ denote the time of entry and $\widetilde{\tau}$ the time of exit from state $s_t = 1$, differentiating both sides of (64) with respect to n_τ and using the envelope theorem gives

$$0 = -V_X^1 Y_t + V_{n_\tau X}^1 \left(r_t X_t^W - C_t^W + \left(\beta \Lambda^d - f_t\left(p_t + \overline{i} + \overline{q}\right) - n_\tau\right) Y_t \right) + V_{n_\tau Y}^1 Y \mu_0 + \frac{1}{2} V_{n_\tau YY}^1 \sigma_Y^2 Y^2 - \left(\rho + \widetilde{\lambda} + g\right) V_{n_\tau}^1.$$

The Feynman-Kac Theorem (see (Karatzas and Shreve 1991) and (Oksendal 1998)) implies that a solution to the above differential equation is given by

$$V_{n_\tau} = -E\left(\int_\tau^{\tilde{\tau}} e^{-\rho(t-\tau)} V_X^1 Y_t dt\right) = -E\left(\int_\tau^{\tilde{\tau}} e^{-\rho(t-\tau)} \left(c^{1,W}\right)^{-\gamma} Y_t^{1-\gamma} dt\right) = -\left(Y_\tau c^{1,W}\right)^{1-\gamma} \varpi \tag{68}$$

The second equality in (68) follows from (65) whereas the third equality follows by using lognormality of Y_t and computing explicitly the associated expectation. Having computed V_{n_τ} we can now return to (63) and take first order conditions with respect to n_t to obtain

$$\pi_t = -\lambda \frac{\overline{A} V_n^1 \left(X_t^W, Y_t; n_t\right)}{V_X^0 Y_t} = \lambda \varpi \frac{\overline{A}\left(c^{1,W}\right)^{-\gamma}}{\left(c^{0,W}\right)^{-\gamma}} \tag{69}$$

Since there are no constraints in the choice of n_t for neither W nor EM, the analog of equation (69) holds for EM as well, so that

$$\pi_t = \lambda \varpi \frac{\overline{A}\left(c^{1,EM}\right)^{-\gamma}}{\left(c^{0,EM}\right)^{-\gamma}}. \tag{70}$$

Finally since $X_t^{EM} = X_t^W = 0$ in the postulated equilibrium, we also know that

$$c^{0,W} = \beta - \left(p^0 + \overline{i}\right) f^0 + \pi n \tag{71}$$

$$c^{1,W} = \Lambda^d \beta - \left(p^1 + \overline{i} + \overline{q}\right) f^1 - n \tag{72}$$

$$c^{0,EM} = 1 + p^0 f^0 - \pi n \tag{73}$$

$$c^{1,EM} = \Lambda^d + p^1 f^1 + n \tag{74}$$

Equations (66), (67), (69),(70), (71)-(74) form a system of ten equations in ten unknowns $(p^j, f^j, \pi, n, c^{j,W}, c^{j,EM})$ for $j = 0, 1$. Plugging (73)-(74) into (69), combining (69) with (70) and using the market clearing condition (12) leads to

$$\frac{c^{1,W}}{c^{0,W}} = \frac{\Lambda^d(\beta+1) - \left(\overline{i} + \overline{q}\right) f^1 - c^{1,W}}{\beta + 1 - \overline{i} f^0 - c^{0,W}} \tag{75}$$

Equation (75) implies that

$$\frac{c^{1,W}}{c^{0,W}} = \frac{\Lambda^d(\beta+1) - \left(\overline{i} + \overline{q}\right) f^1}{\beta + 1 - \overline{i} f^0} \tag{76}$$

Using (76) inside (69) we arrive at equation

$$\pi = \lambda \varpi \frac{\overline{A}\left(c^{1,W}\right)^{-\gamma}}{\left(c^{0,W}\right)^{-\gamma}} = \lambda \varpi \overline{A} \left(\frac{\Lambda^d(\beta+1) - \left(\overline{i} + \overline{q}\right) f^1}{\beta + 1 - \overline{i} f^0}\right)^{-\gamma}, \tag{77}$$

which is precisely equation (18). Using (66) and (77) inside equation (71) leads to

$$c^{0,W} = \beta - \left(p^0 + \overline{i}\right) f^0 + \pi n = \beta - g\frac{\kappa}{\nu} \frac{\left(\beta + \kappa f^0\right)^{-\gamma}}{\left(c^{0,W}\right)^{-\gamma}} f^0 + \lambda \varpi \overline{A} \left(\frac{(\beta+1)\Lambda^d - \left(\overline{i} + \overline{q}\right) f^1}{\beta + 1 - \overline{i} f^0}\right)^{-\gamma} n \tag{78}$$

Solving for n from equation (72), and using (76) and (66) gives

$$n = \Lambda^d \beta - \left(g \frac{\kappa}{\nu} \frac{\left(\beta + \kappa f^1\right)^{-\gamma}}{\left(c^{0,W}\right)^{-\gamma} \left(\frac{\Lambda^d (\beta+1) - \left(\bar{i}+\bar{q}\right) f^1}{\beta + 1 - \bar{i} f^0} \right)^{-\gamma}} \right) f^1 - \left(\frac{\Lambda^d (\beta+1) - \left(\bar{i}+\bar{q}\right) f^1}{\beta + 1 - \bar{i} f^0} \right) c^{0,W} \tag{79}$$

Using (79) inside (78) and rearranging, we arrive at

$$c^{0,W} = \frac{\beta \left[1 + \lambda \varpi \overline{A} \Lambda^d \left(\frac{(\beta+1)\Lambda^d - \left(\bar{i}+\bar{q}\right) f^1}{\beta + 1 - \bar{i} f^0} \right)^{-\gamma} \right] - g \frac{\kappa}{\nu} \left(c^{0,W}\right)^{\gamma} \left(\beta + \kappa f^0\right)^{-\gamma} \left[f^0 + \lambda \varpi \overline{A} \frac{\left(\beta + \kappa f^1\right)^{-\gamma}}{\left(\beta + \kappa f^0\right)^{-\gamma}} f^1 \right]}{1 + \lambda \varpi \overline{A} \left(\frac{(\beta+1)\Lambda^d - \left(\bar{i}+\bar{q}\right) f^1}{\beta + 1 - \bar{i} f^0} \right)^{1-\gamma}} \tag{80}$$

Combining (66) and (67) for $j = 0, 1$ leads to

$$g \frac{\kappa}{\nu} \frac{\left[\kappa \left(1 - f^0\right)\right]^{-\gamma}}{\left(\beta + 1 - \bar{i} f^0 - c^{0,W}\right)^{-\gamma}} = g \frac{\kappa}{\nu} \frac{\left(\beta + \kappa f^0\right)^{-\gamma}}{\left(c^{0,W}\right)^{-\gamma}} - \bar{i} \tag{81}$$

$$g \frac{\kappa}{\nu} \frac{\left[\kappa \left(1 - f^1\right)\right]^{-\gamma}}{\left((\beta+1)\Lambda^d - \left(\bar{i}+\bar{q}\right) f^1 - \upsilon c^{0,W}\right)^{-\gamma}} = g \frac{\kappa}{\nu} \frac{\left(\beta + \kappa f^1\right)^{-\gamma}}{\left(c^{0,W}\right)^{-\gamma} \upsilon^{-\gamma}} - \left(\bar{i}+\bar{q}\right) \tag{82}$$

where

$$\upsilon = \frac{(\beta+1)\Lambda^d - \left(\bar{i}+\bar{q}\right) f^1}{\beta + 1 - \bar{i} f^0}.$$

By solving the system of equations (80), (81) and (82) for $c^{0,W}$, f^0, f^1 we can then substitute into (79) to obtain n, into (77) to obtain π, into (66) to obtain p^0 and p^1 and then inside (71)-(74) to obtain the rest of the allocations.

To complete the verification that the postulated allocation is an equilibrium, we need to show that the consumption processes C_t^W, C_t^{EM} and X_t^{EM}, X_t^W are optimal for EM and W respectively. To show optimality for W it suffices to check that the asserted allocations satisfy the familiar Euler equation:

$$\left(\overline{A}\right)^{N_t} \left(C_t^W\right)^{-\gamma} = E_t \left\{ e^{\left(\int_t^T (r_u - \rho) du\right)} \left(\overline{A}\right)^{N_T} \left(C_T^W\right)^{-\gamma} \right\} \quad \text{for all } t \text{ and } T > t$$

Multiplying both sides of this equation with $e^{\left(\int_0^t (r_u - \rho) du\right)}$ leads to

$$e^{\left(\int_0^t (r_u - \rho) du\right)} \left(\overline{A}\right)^{N_t} \left(C_t^W\right)^{-\gamma} = E_t \left\{ e^{\left(\int_0^T (r_u - \rho) du\right)} \left(\overline{A}\right)^{N_T} \left(C_T^W\right)^{-\gamma} \right\} \text{ for all } t \text{ and } T > t.$$

Hence, an equivalent way to test whether the Euler equation holds is to check whether Z_t, defined as $Z_t = e^{\left(\int_0^t (r_u - \rho) du\right)} \left(\overline{A}\right)^{N_t} \left(C_t^W\right)^{-\gamma}$ is a martingale. Applying Ito's Lemma in state 0 gives

$$\frac{dZ_t}{Z_t} = \left[r^0 - \rho + \lambda \left[\overline{A} \left(\frac{c^{1,W}}{c^{0,W}} \right)^{-\gamma} - 1 \right] + g \left[\left(\frac{\beta + \kappa f^0}{c^{0,W}} \right)^{-\gamma} - 1 \right] - \left(\mu_0 - \frac{\sigma^2}{2} \right) \gamma + \frac{(\gamma\sigma)^2}{2} \right] dt + dM_t \tag{83}$$

where dM_t represents martingale increments. Using equation (17) inside (83) implies that Z_t is driftless and hence the Euler equation holds. A similar argument can be applied to show that the Euler equation holds when $s_t = 1$.

Turning to EM, our assumption that short positions in bonds are subject to the same monitoring costs as growth contingent contracts implies that it is sufficient to check that $X_t^{EM} = 0$ is a "corner" solution for EM. In particular, it suffices to check the following pair of Euler inequalities for all $t < \tau^G$ and all $s_t \in \{0, 1\}$

$$e^{\left(\int_0^t (r_u - \rho) du\right)} \left(\overline{A}\right)^{N_t} \left(C_t^{EM}\right)^{-\gamma} \geq E_t \left\{ e^{\left(\int_t^T (r_u - \rho) du\right)} \left(\overline{A}\right)^{N_T} \left(C_T^{EM}\right)^{-\gamma} \right\} \text{ for all } t \text{ and } T > t. \tag{84}$$

$$e^{\left(\int_0^t (r_u + i_u - \rho) du\right)} \left(\overline{A}\right)^{N_t} \left(C_t^{EM}\right)^{-\gamma} \leq E_t \left\{ e^{\left(\int_0^T (r_u + i_u - \rho) du\right)} \left(\overline{A}\right)^{N_T} \left(C_T^{EM}\right)^{-\gamma} \right\} \tag{85}$$

Equation (84) asserts that EM does not want to invest in bonds at the rate r_t, whereas (85) asserts that borrowing at the rate $r_t + i_t$ is not optimal either. To show equation (84), let τ_-^G denote the instant before development and τ_+^G denote the instant after development. Then for states $j = 0, 1$ equations (81) and (82) imply that

$$\left(\frac{C_{\tau_+^G}^{j,EM}}{C_{\tau_-^G}^{j,EM}}\right)^{-\gamma} + \frac{\nu}{g\kappa}\left(\bar{i} + j\bar{q}\right) = \left(\frac{C_{\tau_+^G}^{j,W}}{C_{\tau_-^G}^{j,W}}\right)^{-\gamma} \Longrightarrow \left(\frac{C_{\tau_+^G}^{j,EM}}{C_{\tau_-^G}^{j,EM}}\right)^{-\gamma} < \left(\frac{C_{\tau_+^G}^{j,W}}{C_{\tau_-^G}^{j,W}}\right)^{-\gamma} \tag{86}$$

For times prior to development ($t < \tau^G$) and any $T > t$ we obtain

$$\begin{aligned} \left(\overline{A}\right)^{N_t} \left(C_t^{EM}\right)^{-\gamma} &= \frac{\left(C_t^{EM}\right)^{-\gamma}}{\left(C_t^{W}\right)^{-\gamma}} \left(\overline{A}\right)^{N_t} \left(C_t^{W}\right)^{-\gamma} = \frac{\left(C_t^{EM}\right)^{-\gamma}}{\left(C_t^{W}\right)^{-\gamma}} E_t \left\{ e^{\left(\int_t^T (r_u - \rho) du\right)} \left(\overline{A}\right)^{N_T} \frac{\left(C_T^{W}\right)^{-\gamma}}{\left(C_T^{EM}\right)^{-\gamma}} \left(C_T^{EM}\right)^{-\gamma} \right\} \\ &\geq E_t \left\{ e^{\left(\int_t^T (r_u - \rho) du\right)} \left(\overline{A}\right)^{N_T} \left(C_T^{EM}\right)^{-\gamma} \right\} \end{aligned}$$

where the first line follows from the fact that W's Euler equation holds as an equality and the inequality follows from the fact that

$$\frac{\left(C_t^{EM}\right)^{-\gamma}}{\left(C_t^{W}\right)^{-\gamma}} \frac{\left(C_T^{W}\right)^{-\gamma}}{\left(C_T^{EM}\right)^{-\gamma}}$$

is either equal to 1 (if $T < \tau^G$) or is larger than 1 (if $T > \tau^G$) by equation (86). To show (85), apply Ito's Lemma and use (17), (20) to obtain

$$\frac{d\left(e^{-\rho t}\left(\overline{A}\right)^{N_t}\left(C_t^{EM}\right)^{-\gamma}\right)}{e^{-\rho t}\left(\overline{A}\right)^{N_t}\left(C_t^{EM}\right)^{-\gamma}} = -r_t dt + g\left(\frac{\left(C_{\tau^{G_+}}^{EM}\right)^{-\gamma}}{\left(C_{\tau^{G_-}}^{EM}\right)^{-\gamma}} - \frac{\left(C_{\tau^{G_+}}^{W}\right)^{-\gamma}}{\left(C_{\tau^{G_-}}^{W}\right)^{-\gamma}}\right) dt + dM_t \tag{87}$$

where dM_t is a martingale increment. Equations (81) and (82) imply that

$$g\left(\frac{\left(C_{\tau^{G_+}}^{EM}\right)^{-\gamma}}{\left(C_{\tau^{G_-}}^{EM}\right)^{-\gamma}} - \frac{\left(C_{\tau^{G_+}}^{W}\right)^{-\gamma}}{\left(C_{\tau^{G_-}}^{W}\right)^{-\gamma}}\right) = -i_t \frac{\nu}{\kappa} \tag{88}$$

Combining (88) and (87) we obtain

$$d\left(e^{\left(\int_0^t (r_u+i_u-\rho)du\right)}\left(\overline{A}\right)^{N_t}\left(C_t^{EM}\right)^{-\gamma}\right) = e^{\left(\int_0^t (r_u+i_u-\rho)du\right)}\left(\overline{A}\right)^{N_t}\left(C_t^{EM}\right)^{-\gamma} i_t\left(1-\frac{\nu}{\kappa}\right)dt + dM_t$$

Assuming that $\nu < 1$, if follows that $e^{\left(\int_0^t (r_u+i_u-\rho)du\right)}\left(\overline{A}\right)^{N_t}\left(C_t^{EM}\right)^{-\gamma}$ is a submartingale and hence (85) holds. This concludes the verification that the postulated consumption process C_t^{EM}, and the asset process $X_t^{EM} = 0$ are optimal for EM. ■

Proof of Lemma 1. Dividing both sides of (80) by β and letting $\beta \to \infty$ shows that $c^{0,W}/\beta \to 1$. Then equation (76) implies that $c^{1,W}/\beta \to \Lambda^d$. Using these facts inside (19), (21) and (18) gives (23). Since $c^{1,W}/c^{0,W} = \Lambda^d = c^{1,EM}/c^{0,EM}$, equations (73), (74) imply that

$$n = \frac{\left(f^0 p^0 - f^1 p^1\right)\Lambda^d}{\left(1+\pi\Lambda^d\right)}. \tag{89}$$

Furthermore, applying equation (67) for $j = 0, 1$ and using $\Lambda^d = c^{1,EM}/c^{0,EM}$ implies that

$$\frac{1}{\Lambda^d}\left(\frac{p^0}{p^1}\right)^{\frac{1}{\gamma}} = \frac{1-f^1}{1-f^0}$$

Since $\frac{1}{\Lambda^d}\left(\frac{p^0}{p^1}\right)^{\frac{1}{\gamma}} > 1$, it follows that $f^0 > f^1$. Rearranging the above equation yields (25). Finally, combining (89) with (25), (73) and (81) yields (24) after straightforward, but tedious manipulations. ■

Proof of Proposition 2. Verifying that the proposed allocations and prices constitute an equilibrium pre-development follows similar steps to the proof of proposition 1. Post development, the value function of both EM and W are the same as in proposition 1. This implies that the value function of agent W is given by equation (62), while the value function of EM is

$$V^G\left(X_{\tau^G}, Y_{\tau^G}\right) = \frac{Y_{\tau^G}^{1-\gamma}}{1-\gamma}\frac{1}{\nu}\left[\kappa\left(1-f_{\tau^G}\right)+\nu x_{\tau^G}\right]^{1-\gamma} \tag{90}$$

Since in the proposed equilibrium all the pre-development equilibrium prices depend exclusively on $\frac{X_t^{EM}}{Y_t} = -\frac{X_t^W}{Y_t}$, the value function of both agents can still be expressed as a function of Y_t, s_t, and X_t^j where $j = \{EM, W\}$. For instance, the Bellman equation for the value function V of agent W in state $s_t = 0$ is

$$\begin{aligned} 0 \;=\; \max_{C_t^W, f_t, \widetilde{\xi}_t} \Bigg\{ & \frac{\left(C_t^W\right)^{1-\gamma}}{1-\gamma} + V_X^0\left(r_t X_t^W - C_t^W + \left(\beta - f_t\left(p_t+\overline{i}\right)+\widetilde{\xi}_t\lambda_t^*\right)Y_t\right) + gV^G\left(X_t^W, Y_t; f_t\right) \\ & + \chi\left[p_{J=1,S=1}\overline{A}V^1\left(X_t^W - \widetilde{\xi}_t Y_t, Y_t\right) + p_{J=1,S=0}V^0\left(X_t^W - \widetilde{\xi}_t Y_t, Y_t\right) + p_{J=0,S=1}\overline{A}V^0\left(X_t^W, Y_t\right)\right] \\ & + V_Y^0 Y \mu_{Y,0} + \frac{1}{2}V_{YY}^0\sigma_Y^2 Y^2 - \left(\rho+\chi+g\right)V^0\Bigg\} \end{aligned} \tag{91}$$

Computing the first order condition for $\widetilde{\xi}_t$ leads to

$$V_X^0\lambda_t^* Y_t = \chi\left[p_{J=1,S=1}\overline{A}V_X^1\left(X_t^W - \widetilde{\xi}_t Y_t, Y_t\right) + p_{J=1,S=0}V_X^0\left(X_t^W - \widetilde{\xi}_t Y_t, Y_t\right)\right]Y_t \tag{92}$$

By combining (92) with the first order condition for consumption ($V_X^0 = \left(C_t^W\right)^{-\gamma}$), using (35), noting that $X_t^{EM} = -X_t^W$ and using the definition of x_t in equation (33) we arrive at (42). The analogous first order condition for EM is

$$\lambda_t^* = \chi \left[p_{J=1,S=1} \overline{A} \left(\frac{K^1(x_t + \widetilde{\xi}_t)}{K^0(x_t)} \right)^{-\gamma} + p_{J=1,S=0} \left(\frac{K^0(x_t + \widetilde{\xi}_t)}{K^0(x_t)} \right)^{-\gamma} \right] \tag{93}$$

By combining (42) and (93) we arrive at (43). As we also explain in the text, equation (43) implies that when $p_{J=1,S=1} = 0$, then $\widetilde{\xi}_t = 0$. Hence, unless there are joint jumps in the marginal utility of consumption and the arrival of jumps in the asset F_t, the optimal holdings of F_t are $\widetilde{\xi}_t = 0$. By assumption, in state $s_t = 1$ there are no joint jumps in the marginal utility of consumption and the jumps in F_t and therefore $\widetilde{\xi}_t = 0$ when $s_t = 1$.

We next turn to the determination of the optimal f^0 and the equilibrium price p^0. Differentiating (91) with respect to f^0, using (35) and (34) and repeating the same steps as in Proposition 1, leads to the first order conditions (38) and (39). Deriving the analogous first order conditions for EM and combining them with (38) and (39) leads to (40) when $s_t = 0$ and to (41) when $s_t = 1$. To simplify notation, we shall denote the solution to (40) as f^0 instead of the more lengthy $f^0(K^0(x_t))$. We will also apply the same shorthand notation to f^1, p^0, p^1, λ^* and $\widetilde{\xi}$. Sofar we have derived all the quantities of interest ($f^0, f^1, p^0, p^1, \lambda^*$ and $\widetilde{\xi}$) as functions of $K^0(x_t)$ and $K^1(x_t)$.

To complete the equilibrium verification, it remains to derive these functions K^0, K^1. We do this by utilizing the Euler equation. Formally, define τ_0 to be the smallest time after time t such that $X_{\tau_0} = 0$:

$$\tau_0 = \inf_{s>t} \{X_s = 0\}$$

Using similar steps as in Proposition 1, we obtain that for any time $t \leq T \leq \tau_0$, the following Euler equation must characterize EM's consumption

$$\overline{A}^{N_t} \left(C_t^{EM}\right)^{-\gamma} = E_t \left\{ e^{(r_t - \rho)(T-t)} \overline{A}^{N_T} \left(C_T^{EM}\right)^{-\gamma} \right\} \tag{94}$$

An identical equation needs to hold for W

$$\overline{A}^{N_t} \left(C_t^{W}\right)^{-\gamma} = E_t \left\{ e^{(r_t - \rho)(T-t)} \overline{A}^{N_T} \left(C_T^{W}\right)^{-\gamma} \right\} \tag{95}$$

Following identical steps to the proof of Proposition 1, one can show that equations (94) and (95) when $s_t = 0$ imply that $e^{(r_t - \rho)t} \overline{A}^{N_t} \left(C_t^W\right)^{-\gamma}$ and $e^{(r_t - \rho)t} \overline{A}^{N_t} \left(C_t^{EM}\right)^{-\gamma}$ are both (local) martingales when $x_t > 0$. Applying Ito's Lemma to x_t gives

$$dx_t = d\left(\frac{X_t}{Y_t}\right) = \frac{dX_t}{Y_t} + X_t d\left(\frac{1}{Y_t}\right) = \alpha(x_t)dt - x_t \sigma dB_t \tag{96}$$

where $\alpha(x_t)$ is defined as

$$\alpha(x_t) \equiv \left(r^0(x_t) - \mu_0 + \sigma^2\right) x_t - K^0(x_t) - \lambda_t^* \widetilde{\xi}_t + 1 + f_t^0 p_t^0 \tag{97}$$

Using (96) and applying Ito's Lemma to $e^{(r_t-\rho)t}\overline{A}^{s_t}\left(C_t^{EM}\right)^{-\gamma}$ when $s_t = 0$ gives

$$\frac{d\left[e^{(r_t-\rho)t}\overline{A}^{N_t}\left(C_t^{EM}\right)^{-\gamma}\right]}{e^{(r_t-\rho)t}\overline{A}^{N_t}\left(C_t^{EM}\right)^{-\gamma}} =$$

$$\begin{aligned} = & \left\{ r^0(x_t) - \rho - \gamma\left(\mu_0 - \frac{\sigma^2}{2}\right) + \frac{\gamma^2\sigma^2}{2} + g\left[\left(\frac{\kappa\left(1 - f_{\tau G}\right) + v x_t}{K^0\left(x_t\right)}\right)^{-\gamma} - 1\right] \right. \\ & + \chi\left[p_{J=1,S=1}\overline{A}\left(\frac{K^1\left(x_t + \widetilde{\xi}_t\right)}{K^0\left(x_t\right)}\right)^{-\gamma} + p_{J=1,S=0}\left(\frac{K^0\left(x_t + \widetilde{\xi}_t\right)}{K^0\left(x_t\right)}\right)^{-\gamma} + p_{J=0,S=1}\overline{A}\left(\frac{K^1\left(x_t\right)}{K^0\left(x_t\right)}\right)^{-\gamma} - 1\right] \\ & \left. - \gamma\left(\frac{\frac{dK^0}{dx_t}}{K^0}\left(\alpha(x_t) - \gamma x_t \sigma^2\right) + \frac{\sigma^2 x_t^2}{2}\frac{\frac{d^2K^0}{(dx_t)^2}}{K^0} - (\gamma+1)\frac{\sigma^2 x_t^2}{2}\left(\frac{\frac{dK^0}{dx_t}}{K^0}\right)^2\right)\right\} dt + dM_t^{EM} \end{aligned} \tag{98}$$

where dM_t^{EM} is a (local) martingale. Since $e^{(r_t-\rho)t}\overline{A}^{N_t}\left(C_t^{EM}\right)^{-\gamma}$ is a local martingale, the term inside the curly brackets in equation (98) must be zero. Combining the definition of α (equation [97]) with the observation that the term inside the curly brackets of (98) is zero allows us to solve for $r^0(x_t)$ as a function of $K^0, K^1, \frac{dK^0}{dx_t}, \frac{d^2K^0}{(dx_t)^2}$ and x_t. We shall denote this function as $r^0(x_t) = r\left(K^0, K^1, \frac{dK^0}{dx_t}, \frac{d^2K^0}{(dx_t)^2}, x_t\right)$. Finally to determine K^0, K^1, we observe that an expression analogous to (98) must also hold for W, namely

$$\frac{d\left[e^{(r_t-\rho)t}\overline{A}^{N_t}\left(C_t^{W}\right)^{-\gamma}\right]}{e^{(r_t-\rho)t}\overline{A}^{N_t}\left(C_t^{W}\right)^{-\gamma}} =$$

$$\begin{aligned} = & \left\{ r^0(x_t) - \rho - \gamma\left(\mu_0 - \frac{\sigma^2}{2}\right) + \frac{\gamma^2\sigma^2}{2} + g\left[\left(\frac{\kappa f_{\tau G} + \beta - v x_{\tau G}}{\Omega^0\left(x_t\right)}\right)^{-\gamma} - 1\right] \right. \\ & + \chi\left[p_{J=1,S=1}\overline{A}\left(\frac{\Omega^1\left(x_t + \widetilde{\xi}_t\right)}{\Omega^0\left(x_t\right)}\right)^{-\gamma} + p_{J=1,S=0}\left(\frac{\Omega^0\left(x_t + \widetilde{\xi}_t\right)}{\Omega^0\left(x_t\right)}\right)^{-\gamma} + p_{J=0,S=1}\overline{A}\left(\frac{\Omega^1\left(x_t\right)}{\Omega^0\left(x_t\right)}\right)^{-\gamma} - 1\right] \\ & \left. - \gamma\left(\frac{\frac{d\Omega^0}{dx_t}}{\Omega^0}\left(\alpha(x_t) - \gamma x_t \sigma^2\right) + \frac{\sigma^2 x_t^2}{2}\frac{\frac{d^2\Omega^0}{(dx_t)^2}}{\Omega^0} - (\gamma+1)\frac{\sigma^2 x_t^2}{2}\left(\frac{\frac{d\Omega^0}{dx_t}}{\Omega^0}\right)^2\right)\right\} dt + dM_t^{W} \end{aligned} \tag{99}$$

where dM_t^W is a local martingale. Since $e^{(r_t-\rho)t}\overline{A}^{N_t}\left(C_t^{W}\right)^{-\gamma}$ is a local martingale, the term inside the curly brackets in equation (99) must be zero. The terms inside the curly brackets of (98) and (99) are both zero, so we can set them equal and use equations (40), (41), and (42), (93) to obtain

$$g\bar{i}\frac{\nu}{\kappa} + \chi p_{J=0,S=1}\overline{A}\left[\left(\frac{\Omega^1\left(x_t\right)}{\Omega^0\left(x_t\right)}\right)^{-\gamma} - \left(\frac{K^1\left(x_t\right)}{K^0\left(x_t\right)}\right)^{-\gamma}\right] = \tag{100}$$

$$= \gamma \left[\left(\frac{\frac{d\Omega^0}{dx_t}}{\Omega^0} - \frac{\frac{dK^0}{dx_t}}{K^0} \right) \left(\alpha(x_t) - \gamma x_t \sigma^2 \right) + \frac{\sigma^2 x_t^2}{2} \left(\frac{\frac{d^2\Omega^0}{(dx_t)^2}}{\Omega^0} - \frac{\frac{d^2K^0}{(dx_t)^2}}{K^0} \right) - (\gamma+1) \frac{\sigma^2 x_t^2}{2} \left(\left(\frac{\frac{d\Omega^0}{dx_t}}{\Omega^0} \right)^2 - \left(\frac{\frac{dK^0}{dx_t}}{K^0} \right)^2 \right) \right]$$

Using equation (36) we obtain

$$\frac{d\Omega^0}{dx_t} = -\left(1 + \bar{i} f_K^0\right) \frac{dK^0}{dx_t} - f_x^0 \tag{101}$$

and

$$\frac{d^2\Omega^0}{(dx_t)^2} = -\bar{i} f_{xK}^0 \frac{dK^0}{dx_t} - \left(1 + \bar{i}(f_K^0 + f_{KK}^0)\right) \frac{d^2K^0}{(dx_t)^2} - f_{xx}^0 \tag{102}$$

By using (36), (101) and (102) inside (100) allows us to obtain an ordinary differential equation involving[26] $K^0, K^1, \frac{dK^0}{dx_t}, \frac{d^2K^0}{(dx_t)^2}$ and x_t. A similar line of derivations allows us to obtain a differential equation in state $s_t = 1$, namely

$$g\left(\bar{i} + \bar{q}\right) \frac{\nu}{\kappa} + \widetilde{\lambda} \left[\left(\frac{\Omega^0(x_t)}{\Omega^1(x_t)} \right)^{-\gamma} - \left(\frac{K^0(x_t)}{K^1(x_t)} \right)^{-\gamma} \right] = \tag{103}$$

$$= \gamma \left[\left(\frac{\frac{d\Omega^1}{dx_t}}{\Omega^1} - \frac{\frac{dK^1}{dx_t}}{K^1} \right) \left(\widetilde{\alpha}(x_t) - \gamma x_t \sigma^2 \right) + \frac{\sigma^2 x_t^2}{2} \left(\frac{\frac{d^2\Omega^1}{(dx_t)^2}}{\Omega^1} - \frac{\frac{d^2K^1}{(dx_t)^2}}{K^1} \right) - (\gamma+1) \frac{\sigma^2 x_t^2}{2} \left(\left(\frac{\frac{d\Omega^1}{dx_t}}{\Omega^1} \right)^2 - \left(\frac{\frac{dK^1}{dx_t}}{K^1} \right)^2 \right) \right]$$

where $\widetilde{\alpha}(x_t) = \left(r^1(x_t) - \mu_1 + \sigma^2\right) x_t - K^1(x_t) + \Lambda^d + f_t^1 p_t^1$. Proving that EM will not choose to borrow in non-contingent bonds when $x_t = 0$ follows similar steps to the proof of Proposition 1. ■

B Data

For the construction of ψ_{it}, we used data from the World Bank's World Development Indicators Database, and from the International Monetary Fund's International Financial Statistics (IFS). Table 8 presents a list of the variables and corresponding sources.

While in the model there is a single good, in the data the computation of ψ_t is more cumbersome since there are multiple goods, exchange rate fluctuations, intermediate goods, and so on. All our steps below are aimed at isolating in ψ_t the component of external resources and income which is transitory in nature. For this, we let:

$$\psi_t = \frac{(\theta_t - 1) Y}{Y} = \frac{\frac{E_t CF_t}{P_{M,t}} + \left[\left(\frac{P_{X,t} X_t}{P_{M,t}} - 0.5 X_t \right) \right]^{Cycle}}{[N_t]^{Trend} + \left[\left(\frac{P_{X,t} X_t}{P_{M,t}} - 0.5 X_t \right) \right]^{Trend}}, \tag{104}$$

where N and X correspond to real nontradables and exports; P_X and P_M to export and import prices in local currency; and E and CF to the nominal exchange rate and capital flows. Real nontradables are constructed from:

$$N_t = \frac{1}{P_{N,t}} \left(GDP_t - (P_{X,t} - 0.5 P_{M,t}) X_t \right)$$

[26] Note also that $\alpha(x_t)$ can be expressed as a function of $K^0, K^1, \frac{dK^0}{dx_t}, \frac{d^2K^0}{(dx_t)^2}$ and x_t since the interest rate is a function of $K^0, K^1, \frac{dK^0}{dx_t}, \frac{d^2K^0}{(dx_t)^2}$ and x_t.

Series	Source
Nominal GDP (GDP)	World Development Indicators (quarterly and annual)
CPI (P)	IFS (quarterly and annual)
Nominal Exports ($P_X X$) (local currency)	World Development Indicators and IFS (quarterly and annual)
Nominal Imports ($P_M M$) (local currency)	World Development Indicators and IFS (quarterly and annual)
Real Exports (X) (local currency)	World Development Indicators (annual)
Real Imports (M) (local currency)	World Development Indicators (annual)
Nominal Capital Flows (CF) (dollars)	IFS (quarterly and annual)
Nominal Exchange Rate (E)	World Development Indicators and IFS (quarterly and annual)
Net Factor Payments (NFP) (dollars)	IFS (annual)

Table 8: Data used in the construction of ψ.

	Mean	Median	Min	Max
Chile	20.6	21.5	15.4	22.7
Colombia	11.3	10	8.4	16.6
Mexico	5.5	5.6	2.7	7.2
Indonesia	11.3	7.4	4.7	19.8
Malaysia	29.4	28.6	19.6	42.3
Thailand	21.3	20.8	14.1	28

Table 9: Reserves for various countries as a percent of GDP for the years 1990-2003.

where GDP is the country's GDP, $P_{N,t}$ is the price of nontradables approximated by the local CPI, and the term $0.5P_{M,t}$ removes a proxy for intermediate inputs in export-production. The expression $\left(\frac{P_{X,t}X_t}{P_{M,t}} - 0.5X_t\right)$ captures the terms of trade effect. We decompose between trends and cycles using a standard Hodrick-Prescott filter, extending the series as much as we could in order to reduce the effect of the end-of-series bias in this procedure. We applied the filter to the log of the corresponding variable. In summary, the denominator in equation (104) measures the average (trend level) of total income and resources, while the numerator attempts to capture the cyclical component of *external* resources.

We work with a sample of the following developing countries/emerging markets: Chile, Colombia, Indonesia, Malaysia, Mexico, Thailand. We chose them from the list of countries in Calvo, Izquierdo and Mejia (2004) plus Malaysia. However, since their sample is for the 1990s only, and we needed a longer time series dimension (we used data from 1983 to 2003), we dropped all the countries that either were closed economies with domestic macroeconomic issues, or did not have complete data. Our marginal drop was Korea, for which we did not have good deflators. However, when we re-estimated our model with Korea included (using only capital flows data divided by nominal GDP for Korea), our results remained essentially unchanged. Finally, we constructed quarterly series for $\widetilde{\psi}_{it}$ using a related series approach with quarterly data on capital flows. We restrict the average of the quarterly values to be equal to the annual figure we computed directly using equation (104). Unfortunately, we lack quarterly data for some components of ψ_t for some countries in the 1990-2003 period. To solve this problem we use a linear (or a quadratic) interpolation method to obtain quarterly series for the years when we lack quarterly data. We used interpolation methods for the following countries (dates are shown in parenthesis): Chile (1990), Colombia (1990-1995), and Malaysia (1990-1998).

As a reference, table 9 shows reserves for the six economies we study.

C Details on the econometric procedure of Sections 3.1, 3.2.1, 3.2.2.

Section 3.1: To estimate the process described in this section we apply a Bayesian methodology, by using a Gibbs Sampler. The Gibbs Sampler is by now a standard methodology in estimating models involving hidden states (See Kim and Nelson (1999) for an introductory treatment). The basic idea is to exploit knowledge about the conditional distribution of one parameter at a time (fixing all the others) to construct the joint posterior distribution of all parameters. We modify the basic model that Kim and Nelson (1999) present by pooling all the countries into a single sample. We allow all parameters of the model to differ across countries. However, in order to obtain precise estimates we assume that the transition probabilities into and out of a sudden stop are *the same* across countries. Moreover, we assume that the *joint probability* of a jump in the VIX and a simultaneous transition into a sudden stop is *common* across countries.

The first step of the procedure is to fix a set of initial parameters $\Psi = \{\psi_i^{NSS}, \psi_i^{SS} - \psi_i^{NSS}, \sigma_{e,i}^{NSS}, \sigma_{e,i}^{SS}, p(NSS \rightarrow SS), p(SS \rightarrow NSS)\}$ and then determine the posterior probabilities that a particular realization of $\widetilde{\psi}_{it}$ for country i at time t was drawn from the first (NSS) or the second (SS) regime. To do that we run a standard Hamilton (1989,1990) type filter as described in Kim and Nelson (1999) to determine a sequence of posterior probabilities that a given country was in a sudden stop at a specific point in time. We repeat this process for each country separately and obtain one sequence per country. We shall denote this as $\Pr(SS = 1|\widetilde{\psi}_i; \Psi)$. In the next step we draw an (artificial) sample of 1's and 0's from these posterior probabilities. As in the text, we use the convention that 1 corresponds to a Sudden Stop and 0 to NSS.

In the next step we take these 1's and 0's as given. Effectively this allows us to proceed *as if we knew* whether each economy is in SS or not at a given point in time. Then we use this information to determine the posterior distributions of the parameters in Ψ. Once again we do this in steps as described in Kim and Nelson (1999). We start with determining the posterior distribution of $\{\psi_i^{NSS}, \psi_i^{SS} - \psi_i^{NSS}, \sigma_{e,i}^{NSS}, \sigma_{e,i}^{SS}\}$ first: To facilitate the updating we use conjugate priors: a) a beta prior for $p(NSS \rightarrow SS)$, $p(SS \rightarrow NSS)$ with $\alpha = \beta = 1$ which coincides with a uniform prior on $[0, 1]$ b) an (improper) normal prior for ψ_i^{NSS} and an (improper) inverse gamma prior for $\left(\sigma_{e,i}^{NSS}\right)^2$ that lead to posteriors that depend only on the data (see Kim and Nelson (1999)) c) a truncated (improper) normal and an inverse (improper) gamma prior for $\psi_i^{SS} - \psi_i^{NSS}$, $\left(\sigma_{e,i}^{SS}\right)^2$ as explained in Kim and Nelson (1999). Finally, we assume that all priors are independent of each other. By well known results in Bayesian statistics the posterior distributions are in the same class as these conjugate priors and there are simple closed form expressions for the parameters of the posterior distributions. The updating of $p(NSS \rightarrow SS)$,and $p(SS \rightarrow NSS)$ is done by pooling the observations for all the countries. So, for each country we count the number of transitions into and out of

sudden stops, and the total number of periods in normal times and in sudden stops. We then add all the episodes for all countries and find the posterior distributions as follows

$$p(NSS \rightarrow SS) \sim beta\left(1 + \sum_i a_i^{SS}, 1 + \sum_i a_i^{NSS}\right)$$

where a_i^{SS} is the number of observations marked as normal years that are followed by a year marked as sudden stop. We shall refer to this as a *transition to a sudden stop*. Conversely, a_i^{NSS} counts the times that a normal year is followed by another normal year (NSS). This count is done country by country, but then we add all them up into a single number which is used in the updating process. By restricting the transition probabilities to be the same for all countries, we exploit the panel dimension of the data in order to obtain precise estimates. Similarly, the posterior for the other parameter in the transition matrix is given by the following formula

$$p(SS \rightarrow NSS) \sim beta\left(1 + \sum_i b_i^{NSS}, 1 + \sum_i b_i^{SS}\right)$$

where b_i^{NSS} is the number of observations marked as sudden stops that are followed by a year marked as "normal". Conversely, b_i^{SS} represents the other case, namely when a transition out of a sudden stop does not occur. We record the random draws of a) the paths of 1's and 0's for each country, b) the country specific parameters $\{\psi_i^{NSS}, \psi_i^{SS} - \psi_i^{NSS}, \sigma_{e,i}^{NSS}, \sigma_{e,i}^{SS}\}$ and c) the "pooled estimates" of $p(NSS \rightarrow SS)$,and $p(SS \rightarrow NSS)$. Then we repeat the above procedure several times and at each time we record the new draw of the paths of 1's and 0's for each country and the parameters. By properties of the Gibbs sampler, the posterior distribution of these random draws coincides in law with the posterior (joint) distribution of all the parameters.

Section 3.2.2: By using the cutoff value $\overline{e}$ for the VIX we determine the months in which we observed a *"jump"* in the VIX, i.e. months when the residuals of the estimated AR(1) process of the VIX exceeded the cutoff value $\overline{e}$. To determine a distribution of $p(SS|J)$ we proceed as follows. First, for each country we draw paths from the posterior distribution of the states $(NSS, SS)_i$, as provided by the Gibbs Sampler. Given the model, the only relevant observations for the conditional probability are the ones when the country i is in NSS or has just transitioned to a SS. Hence, in accordance with the data generating process of the model, we discard all the quarters in which the country is in SS, except the one that marks the beginning of each SS. Then we look at all those times where the states switch from NSS to SS and simultaneously there is a jump in the VIX either in that quarter or the quarter before. We allow this short window to allow for the possibility of delayed data reporting etc. Let this number be given by $n^i_{NSS \rightarrow SS,J}$. Similarly, we also determine all the times when there was a jump in the VIX. Let this number be n^i_J. Finally, define n^i_{NSS} as the numbers of observations when country i is either in NSS or just moved to a SS. We repeat

this procedure for each country separately. After completing the above procedure for all countries we sum $n^i_{SS,J}$, n^i_J, and n^i_{NSS} across countries. In accordance with the Bayesian methodology that we have been using throughout, we use a beta distribution with uniform priors to obtain the posterior distributions

$$\begin{aligned} p(J\ ) &\sim beta\left(1+\Sigma_i n^i_J, 1+\Sigma_i n^i_{NSS}\right) \\ p(SS|\ J) &\sim beta\left(1+\Sigma_i n^i_{NSS\to SS,J}, 1+\Sigma_i n^i_J\right) \end{aligned}$$

In a nutshell, for each iteration of the Gibbs sampler, we determine the number of times when a jump in the VIX coincided with a transition into a SS for each country. Then we pool across countries. This effectively imposes the constraint that all countries have the same joint probability distributions between transitions into a SS and jumps in the VIX. The benefit is that we can obtain more accurate estimates[27].

D Estimation of Risk Premia

To estimate the risk premium λ^* we followed an approach similar to (Ait-Sahalia and Lo 2000), which itself builds on the ideas of (Breeden and Litzenberger 1978) and Duffie and Huang (1985). As explained in the text, if $P^C(VIX,K,\tau)$ is the price of a call option with strike price K and τ days to expiration, when the underlying value of the index is VIX, then the (arbitrage free) price of a payoff such as (50) is simply $-\frac{dP^C(VIX,K,\tau)}{dK}$. To estimate this quantity we fit first a non-parametric function $P^C(VIX,K,\tau)$ to the data. We used quotes[28] from the CBOE on VIX call options since the beginning of this market in early 2006. Letting $k=\frac{K}{VIX}$, we then estimated the following regression:

$$\begin{aligned} \frac{P^C(VIX,K,\tau)}{VIX} &= 0.0306K - 1.115k + 0.34k^2 - 0.0535k^3 + 0.0024k^4 \\ &\quad -0.0002\tau K - 0.0304\tau k + 0.0243\tau k^2 - 0.008\tau k^3 + 0.001\tau k^4 + \ .. \\ &\quad \text{... additional controls} \end{aligned} \tag{105}$$

[27] As a robustness check we also computed $p(J\ )$ and $p(SS|\ J)$ without imposing any prior, i.e. by just recording the random draws of:

$$\begin{aligned} p^{(k)}(J) &= \frac{\Sigma_i n^i_J}{\Sigma_i n^i_{NSS}} \\ p^{(k)}(SS|\ J) &= \frac{\Sigma_i n^i_{NSS\to SS,J}}{\Sigma_i n^i_J} \end{aligned}$$

at each iteration k of the Gibbs Sampler and computing the empirical mean of the corresponding stationary distribution. The two approaches delivered very similar results, suggesting that our results are not influenced by the assumption of a uniform prior.

[28] In particular, we used the midpoint between bid and ask

where additional controls included a $1/VIX$, a constant, VIX, τ, τ^2. To avoid micro-structural problems with very short maturities,[29] we focused attention on values of $\tau \in [30, 90]$. Furthermore, since we are interested in isolating the price of risk for upward jumps in the VIX we focused on $k > 1$. The regression was estimated on 2476 date-price combinations. The resulting R^2 of the regression was 0.83, which suggests that the regression specification accounts well for the observed variation in call option prices. We then differentiated the estimated $P^C(VIX, K, \tau)$ in equation (105) with respect to K . Since we focus attention on short-dated options but with expiry dates larger than 30 days, we evaluated $-\frac{dP^C(VIX,K,\tau)}{dK}$ at $\tau = 40$, and at the average value of VIX over the sample in order to make sure that τ is an interior value to the regression (105). To be consistent with the definition of "jumps" that we used in the text we set

$$k = \overline{e}\sqrt{\frac{\tau}{30}} + 0.17\frac{\tau}{30}\left(\overline{\overline{\log(VIX)}} - \overline{\log(VIX)}\right).$$

The first term in k is set so that the probability of having a jump in the VIX residuals-assuming no jump over an interval τ- is approximately .01. The second term accounts for mean reversion in VIX by comparing the distance between the average (log) underlying price in the call options $\overline{\log(VIX)} = 2.54$ to its long run mean $\overline{\overline{\log(VIX)}} = 2.88$ (based on data since 1986). The coefficient 0.17 accounts for the speed of mean reversion in monthly data. Plugging in these numbers, we obtain that the value of a claim such as (50) is 0.105. As was shown in the text, this number implies that $\lambda^*(\tau/360) = 0.105$. Solving for λ^* gives 0.94.

[29] The data are quoted at prices with discrete increments. For the out of the money call options that we consider, this discreteness presents a problem at very short intervals. As maturity nears to less than thirty days a large fraction of the out of the money call options start to converge to the lowest possible increment of bid and the next highest increment of ask. This discreteness is less of a problem for longer maturities. This is why we leave out the very short maturities.

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